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Landmark VII-C A mixed-Wick obstruction to a normal-topology boundary extension for sharp Cauchy harmonic insertions

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Abstract

We retain the sharp zero-Cauchy harmonic dressing and its exact off-shell classical functional, and examine a boundary enlargement of the free polynomial Wick domain. The outcome is a specified continuity obstruction, not an algebra construction. For the fixed Minkowski de Donder two-point matrix, the full tensor calculation gives $H_{ww} = 0$, while H_{wh} contains a Cauchy-reflected frequency branch. Linear products, the insertion self-product, and a mixed quadratic test admit compact distributional polynomial representatives. Nevertheless, the mandatory ordered word $B_f \star W_g \star B_f$, with $W_g = \int g\, h_{00}^2$, has a fully contracted coefficient whose regular-kernel approximations grow logarithmically. We compute the nonzero coefficient explicitly as a product of reflected light-ray transforms. The approximating insertion kernels converge in a specified normal topology, so this divergence rules out a sequentially continuous extension of that coefficient. Consequently, it rules out a hypocontinuous Wick algebra extension satisfying the stated admission and coefficient-continuity requirements. The logarithmic coefficient can be nonzero for three pairwise disjoint observation supports. No free equation is imposed on configurations, no additional coordinate field is introduced, and no retarded product is constructed.

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Problem, fixed model, and precise scope

boxed VII-C changes the insertion domain only.

The sharp dressing considered here is

$$L = \mathrm{E}_0 C, \qquad w[h] = Lh, \qquad B_f[h,\varphi] = -\int_M f(x) w^{\mu}[h](x) \partial_{\mu}\varphi(x)\, \mathrm{d}^4x.$$

The minus sign agrees with the first geometric coefficient $\mathscr{B}_f^{(1)}$. All classical configurations are smooth and unrestricted by their equations of motion.

The preceding obstruction is not interpreted as closing the interacting product problem. Here we ask a narrower question: does admitting the actual compact distributional kernel of , with a specified normal topology, permit a continuous extension of the fixed free Wick product?

The answer proved below is negative for a named mandatory graph. The continuity assumptions are part of the theorem. An abstract algebra with a different topology, a smaller collection of bulk observables, or additional renormalized boundary operations is not excluded.

Conventions

Let

$$M = \mathbb{R}^{1+3}, \qquad \eta = \mathrm{diag}\,(-1,1,1,1), \qquad \mathrm{square} = -\partial_t^2 + \Delta_{\mathbf{x}}, \qquad \Sigma_0 = \{t = 0\}, \qquad M_+ = \{t > 0\}.$$

Indices are raised with η, and

$$(Ch)^\mu = \partial_\nu h^{\mu\nu} - \frac{1}{2}\partial^\mu(\eta^{\alpha\beta}h_{\alpha\beta}).$$

The zero-Cauchy inverse satisfies

$$\mathrm{square}\,\mathrm{E}_0 j = j, \qquad (\mathrm{E}_0 j)|_{\Sigma_0} = 0, \qquad \partial_t(\mathrm{E}_0 j)|_{\Sigma_0} = 0.$$

We take $f \in C_c^\infty(M_+)$. Thus its support has a positive distance in the time coordinate from Σ_0.

Signature dictionary. Relative to the convention $\eta_{\mathrm{B}} = \mathrm{diag}\,(1,-1,-1,-1)$,

$$\eta = -\eta_{\mathrm{B}}, \qquad \mathrm{square} = -\mathrm{square}_{\mathrm{B}}, \qquad \mathrm{E}_0 = -\mathrm{E}_0^{\mathrm{B}}.$$

All raised-index expressions are recomputed using η. If one also changes $h = -h_{\mathrm{B}}$ so that the full metric changes sign, then $C[h] = -C_{\mathrm{B}}[h_{\mathrm{B}}]$ and $\mathrm{E}_0 C[h] = \mathrm{E}_0^{\mathrm{B}} C_{\mathrm{B}}[h_{\mathrm{B}}]$.

The fixed free matrix

The active even blocks are

$$H_{hh\,\alpha\beta\rho\sigma}(x,y) = P_{\alpha\beta\rho\sigma}\Delta_0^+(x-y),$$

$$P_{\alpha\beta\rho\sigma} = \eta_{\alpha(\rho}\eta_{\sigma)\beta} - \frac{1}{2}\eta_{\alpha\beta}\eta_{\rho\sigma},$$

$$H_{\varphi\varphi}(x,y) = \Delta_m^+(x-y), \qquad H_{h\varphi} = H_{\varphi h} = 0,$$

where $m \geq 0$ is fixed and

$$\Delta_m^+(x-y) = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 p}{(2\pi)^3\, 2\omega_m(\mathbf{p})} e^{-i\omega_m(\mathbf{p})(t-s) + i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}, \qquad \omega_m(\mathbf{p}) = \sqrt{|\mathbf{p}|^2 + m^2}.$$

No positivity is assumed for the graviton block.

The remaining gauge-fixed multiplet components are spectators in every calculation below. An enlargement of the full multiplet algebra must, in particular, admit the displayed even-sector products. Therefore an obstruction in this sector is sufficient for the stated conclusion. No inference about physical observables in gauge cohomology is made; that is a separate problem in the perturbative gravitational framework [5].

We use the Wick-symbol convention

$$F \star_H G = \sum_{r \geq 0} \frac{\hbar^r}{r!} \langle F^{(r)}, H^{\otimes r} G^{(r)} \rangle.$$

For finite polynomials this sum is finite. A local expression such as $\int g h_{00}^2$ denotes its Wick symbol; no self-contraction is included inside that expression.

The old polynomial domain

Let $\mathcal{P}_{\mathrm{mc}}$ denote finite polynomial functionals with compact distributional kernels satisfying the usual microcausal exclusion: for a kernel of degree n,

$$\mathrm{WF}(u_n) \cap \left(\widetilde{\overline{V}}_+^{\,n} \cup \widetilde{\overline{V}}_-^{\,n} \right) = \varnothing, \qquad \widetilde{\overline{V}}_\pm = \overline{V}_\pm \cup \{0\},$$

with the total zero covector omitted. Let $\mathcal{P}_{\mathrm{reg}} \subset \mathcal{P}_{\mathrm{mc}}$ denote the finite polynomials with smooth compact kernels.

Only finite polynomials are used. Results about unrestricted smooth microcausal functionals are not presumed. In particular, the distinction between polynomial and general functional domains identified by Hawkins, Rejzner, and Visser is respected [3]. Their generalized-cone framework motivates a cone test, but supplies no premise asserting that B_f passes it.

What will be proved

The principal result is the following statement, whose topology and coefficient are defined precisely in Sections and .

Theorem (Main result: a specified C-obstruction). *For the fixed matrix –, there exist real nonnegative* $f, f', g \in C_c^\infty(M_+)$*, with pairwise disjoint supports, and regular polynomials* $B_{f,\varepsilon}, B_{f',\varepsilon}$ *such that:*

1. *their mixed kernels converge to the exact kernels of* $B_f, B_{f'}$ *in the normal-topology stages specified in Section ;*

2. *with* $W_g = \int g(y) h_{00}(y)^2 \, \mathrm{d}^4 y$*,*

$$\mathrm{ev}_0 [\hbar^3] \big(B_{f,\varepsilon} \star_H W_g \star_H B_{f',\varepsilon} \big) = \mathscr{C}(f, g, f') \log(\varepsilon^{-1}) + O(1),$$

where $\mathscr{C}(f, g, f') > 0$*;*

3. *the corresponding fully contracted coefficient has no sequentially continuous extension at the pair of sharp kernels that agrees with the old Wick product on regular kernels. In particular, no algebra extension satisfying Assumption exists.*

The evaluation ev_0 sets all classical fields to zero and hence extracts the field-independent coefficient. It is not a replacement of the algebra-valued target by a matrix element: it is a necessary continuous coefficient map of the proposed algebra extension.

Exact off-shell reconstruction and causal support

Write

$$\gamma_0 h = h|_{\Sigma_0}, \qquad \gamma_1 h = \partial_t h|_{\Sigma_0}, \qquad j = \mathrm{square}\, h.$$

All formulas act componentwise on h.

Proposition (Off-shell reconstruction). *For every smooth* h*, on* M_+*,*

$$h = \mathrm{U}(\gamma_0 h, \gamma_1 h) + \mathrm{E}_0 j,$$

$$w[h] = \mathrm{E}_0 C \mathrm{U}(\gamma_0 h, \gamma_1 h) + \mathrm{E}_0 C \mathrm{E}_0 j.$$

Consequently,

$$B_f = B_{f,\,\mathrm{data}} + B_{f,\,\mathrm{source}}$$

is an exact off-shell identity.

For $K = \operatorname{supp} f$*, set*

$$\mathscr{D}_K = J^-(K) \cap \{0 \le t \le \max_{x \in K} t(x)\}.$$

Then $\mathscr{D}_K$ *is compact, and* B_f *depends on h only in* $\mathscr{D}_K$ *and on φ only in K.*

Proof. Let $A = \sqrt{-\Delta_{\mathbf{x}}}$. In spatial Fourier notation,

$$\mathrm{U}(a, b)(t) = \cos(tA)a + \frac{\sin(tA)}{A} b,$$

and

$$(\mathrm{E}_0 j)(t) = -\int_0^t \frac{\sin((t-u)A)}{A} j(u)\,\mathrm{d}u.$$

The quotients at zero spatial frequency are interpreted by their smooth limits.

The difference between the two sides of solves the homogeneous wave equation with zero Cauchy data, and therefore vanishes. Applying $\mathrm{E}_0 C$ gives , and substituting into gives the claimed split.

Finite propagation speed implies that $w[h](x)$, for $x \in K$, depends only on the restriction of h to the causal segment between Σ_0 and x. The same holds for its derivatives. The union of these segments is contained in $\mathscr{D}_K$.

Compactness follows directly from compactness of K and the Minkowski causal bound.

Remark (The source is retained). The triple $(\gamma_0 h, \gamma_1 h, j)$ is a presentation of the same off-shell field, not a new collection of propagating fields. In particular, j is not set to zero.

For linear source smearings, the bisolution identities imply

$$H_{jh} = 0, \qquad H_{jj} = 0.$$

Thus such source variables are central for the free Wick product. This does not make them zero classical functionals, and it does not by itself define products involving arbitrary singular source-sector kernels.

The full insertion kernel and the characteristic-cone test

A joint wavefront upper bound

We order the mixed kernel variables as (x, z), with x the scalar slot and z the graviton slot. There is a compact tensor-valued distribution $b_f(x, z)$ such that

$$B_f[h, \varphi] = \langle b_f, \varphi \otimes h \rangle.$$

The compact-kernel assertion follows either from the explicit construction below or from continuity of the bilinear functional and Proposition .

Let E_{ret} denote the retarded fundamental solution of square. For $x^0 > 0$,

$$(Lh)(x) = \int E_{\mathrm{ret}}(x - z)\theta(z^0)(Ch)(z)\, \mathrm{d}^4 z.$$

The identity is local in x, so harmless compact cutoffs can be inserted in the integration variables.

For covectors in the fixed Cartesian coordinates, define

$$\Lambda_{\mathrm{diag}} = \{(\xi, \zeta) \neq 0 : \zeta = -\xi\},$$

$$\Lambda_{\partial} = \{(\xi, \zeta) \neq 0 : \eta^{-1}(\xi, \xi) = 0,\ \xi + \zeta \in \mathbb{R}\ \mathrm{d}t\},$$

$$\Lambda_B = \Lambda_{\mathrm{diag}} \cup \Lambda_{\partial}, \qquad \Gamma_B = (M \times M) \times \Lambda_B.$$

The case $\xi = 0$ is included in Λ_{∂}.

Lemma (Joint-kernel control). *The kernel b_f satisfies*

$$\mathrm{WF}\,(b_f) \subset \Gamma_B.$$

More precisely, its possible singular branches arise from:

1. *the bulk diagonal* $x = z$*;*

2. *direct null propagation from* z *to* x*;*

3. $z^0 = 0$ *together with null propagation, with covectors*

$$(\xi, -\xi + \tau\ \mathrm{d}t);$$

4. *pure boundary conormals*

$$(0, \tau\ \mathrm{d}t).$$

This is an upper bound, not an assertion that every listed covector occurs.

Proof. The retarded wave kernel has diagonal conormals and, away from the diagonal, the null propagation relation $(x, \xi; z, -\xi)$. Multiplication by $\theta(z^0)$ is permitted: a wavefront covector of that factor has zero x-component, whereas a wavefront covector of the retarded kernel has nonzero x-component. Hence opposite covectors cannot sum to zero.

The product theorem yields the direct branches, the boundary sums $(\xi, -\xi + \tau\ \mathrm{d}t)$, and the pure boundary conormals. At $z^0 = 0$, the bulk diagonal does not meet the support of f, because f is supported strictly in M_+.

Integrating the first-order operator C by parts in z gives the kernel of L. This can produce a $\delta(z^0)$, but not a $\delta'(z^0)$. Finally, moving ∂_μ from φ onto the x-kernel and multiplying by $f(x)$ do not enlarge the wavefront set. This proves the assertion.

The distinctions between multiplication, pullback, and pushforward used here are the standard distributional ones; membership in a wavefront-restricted vector space is not itself an operation theorem [2].

The characteristic-cone enlargement does not admit the insertion

Let $\mathcal{N}_\pm$ be the two null cones in the cotangent bundle and put $\widetilde{\mathcal{N}}_\pm = \mathcal{N}_\pm \cup \{0\}$. The proposed characteristic-cone

condition is

$$\mathrm{WF}(u_n) \cap \left(\widetilde{\mathcal{N}}_+^n \cup \widetilde{\mathcal{N}}_-^n \right) = \varnothing.$$

Proposition (Failure of the characteristic-cone test). *For every nonzero $f \in C_c^\infty(M_+)$,*

$$B_f \notin \mathcal{P}_{\mathrm{char}}.$$

In particular, its full mixed kernel cannot be admitted merely by replacing the causal exclusion cones by the null exclusion cones.

Proof. Fix $\mathbf{n} \in S^2$, let $r > 0$, and consider the smooth complexified configuration

$$h^{(r)}_{\alpha\beta}(z) = P_{\alpha\beta 00} e^{-irz^0 + ir\mathbf{n}\cdot\mathbf{z}}.$$

A direct use of gives

$$(Ch^{(r)})^\mu = -ir\,\delta^\mu_0 e^{-irz^0 + ir\mathbf{n}\cdot\mathbf{z}}.$$

Solving the zero-Cauchy equation yields

$$w^0[h^{(r)}](t,\mathbf{x}) = \left(-\frac{t}{2} e^{-irt} + \frac{\sin(rt)}{2r} \right) e^{ir\mathbf{n}\cdot\mathbf{x}}, \qquad w^i[h^{(r)}] = 0.$$

Choose $a \in C_c^\infty(M_+)$ with $\int f a \, \mathrm{d}^4x \neq 0$, and set

$$\varphi_r(x) = a(x) e^{-irt - ir\mathbf{n}\cdot\mathbf{x}}.$$

Substitution into gives

$$B_f[h^{(r)}, \varphi_r] = \frac{1}{4} \int f a \, \mathrm{d}^4x + O(r^{-1}).$$

Indeed, the nonoscillating term is obtained from

$$\sin(rt) e^{-irt} = \frac{1 - e^{-2irt}}{2i};$$

all terms containing e^{-2irt} decrease rapidly after the t-integration, including the term with the prefactor rt. The term containing $\partial_t a$ is $O(r^{-1})$.

This is a Fourier test of the compact kernel $a(x)b_f(x,z)$, contracted with the constant tensor $P_{\alpha\beta 00}$, along a ray for which both slot covectors are null and have the same temporal orientation. If the characteristic-cone condition held, compactness and the definition of the wavefront set would imply rapid decrease along that ray. The nonzero limit contradicts it.

If desired, the plane wave in the graviton slot can be multiplied by a compact cutoff equal to one on a neighborhood of $\mathscr{D}_{\operatorname{supp} f}$, without changing the calculation.

Cauchy covariances and the full tensor audit

Cauchy pullbacks

Define

$$H_\Sigma^{ij} = (\gamma_i \otimes \gamma_j) H_{hh}, \qquad i,j \in \{0,1\}.$$

These pullbacks exist because the conormals of the spacelike Cauchy surface are timelike, whereas the nonzero Hadamard covectors are null. The same argument permits the bulk–boundary blocks. This is an application of the pullback criterion, not a multiplication of boundary delta distributions [2].

For the scalar massless factor, spatial Fourier transformation gives

$$\widehat{H}_\Sigma(\mathbf{p}) = \frac{1}{2r}\begin{pmatrix} 1 & ir \\ -ir & r^2 \end{pmatrix}, \qquad r = |\mathbf{p}|,$$

multiplied by $P_{\alpha\beta\rho\sigma}$ in the graviton sector. The entries are locally integrable at $r = 0$ in three spatial dimensions. The full matrix, including the off-diagonal phases, is retained.

The vanishing ww contraction

Proposition (Tensor cancellation). *For the matrix ,*

$$C_x^{\mu} H_{hh\,\rho\sigma}(x,y) = \delta^{\mu}_{(\rho}\partial^x_{\sigma)}\Delta_0^+(x-y),$$

$$C_x^{\mu} C_y^{\nu} H_{hh}(x,y) = -\frac{1}{2}\eta^{\mu\nu}\mathrm{square}_x\,\Delta_0^+(x-y) = 0.$$

The induced free Cauchy contractions satisfy

$$H_{ww} = 0.$$

Proof. In four dimensions,

$$\eta^{\alpha\beta} P_{\alpha\beta\rho\sigma} = -\eta_{\rho\sigma}.$$

The trace contribution in C_x^{μ} cancels the trace term in the first divergence of P, leaving . Applying C_y^{ν}, using $\partial^y = -\partial^x$ on $\Delta_0^+(x-y)$, cancels the two mixed second-derivative terms and gives .

To justify the Cauchy operations without multiplying singular unsmeared kernels, use . On each massless momentum mode, the twice-contracted tensor numerator in vanishes before either zero-Cauchy solution operator is applied. The remaining mode factors have polynomial growth and locally integrable infrared behavior. Thus the smeared contraction is zero. Equivalently, the Cauchy covariance construction based on gives the zero distribution.

Remark (Three limitations). The preceding result does not say that $w[h]$ is the zero off-shell functional. It does not say that $H_{wh} = 0$. It depends on the fixed matrix ; no claim is made for an arbitrary gauge choice or an arbitrary smooth change of covariance.

The mixed covariance and its reflected branch

The component needed for the obstruction is

$$K(x,y) := H^0_{wh00}(x,y).$$

By ,

$$K(x,y) = \mathrm{E}_0^x \partial_t \Delta_0^+(x-y).$$

For $r > 0$,

$$\mathrm{E}_0(e^{-irt}) = -\frac{it}{2r}e^{-irt} + \frac{i\sin(rt)}{2r^2}.$$

Consequently,

$$K(x,y) = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 p}{(2\pi)^3} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}\Big[\Big(-\frac{t}{4r} + \frac{i}{8r^2}\Big)e^{-ir(t-s)} - \frac{i}{8r^2}e^{ir(t+s)}\Big].$$

The terms r^{-1} and r^{-2} are locally integrable at $\mathbf{p} = 0$. At large momentum this is an oscillatory distribution with symbol amplitudes.

The last term in is the *Cauchy-reflected branch*. Its first time frequency is opposite to that of the direct Hadamard branch, while its second time frequency is unchanged. This is the branch responsible for the obstruction.

Reality of $\mathrm{E}_0 C$ and Hermiticity of the fixed covariance imply

$$H_{hw\,00}{}^{0}(y,x) = \overline{K(x,y)}.$$

The finite tests that do pass

The results in this section are existence statements for specified compact polynomial kernels. They are not a closure theorem.

Linear generators and insertion–linear products

Independently smeared Cauchy fields are admitted using . Their pairings with ordinary spacetime smearings are well defined. This does not define coincident products of boundary fields.

For a compact smooth graviton test tensor a, write

$$h(a) = \int a^{\alpha\beta} h_{\alpha\beta}.$$

Since $H_{hh}a$ is smooth,

$$B_{f \star_H} h(a) = B_f h(a) - \hbar \int f (L(H_{hh}a))^{\mu} \partial_{\mu} \varphi.$$

Likewise, for $u \in C_c^\infty(M)$,

$$B_f \star_H \varphi(u) = B_f\varphi(u) - \hbar \int f\, w^\mu[h]\, \partial_\mu(H_{\varphi\varphi} u).$$

The reverse products use the transposed covariance. All displayed coefficients are compact distributional polynomial functionals with the causal support inherited from Proposition .

The insertion self-product

Proposition (A well-defined self-product kernel). *For $f, f' \in C_c^\infty(M_+)$, the following compact distributional polynomial is well defined:*

$$B_f B_{f'} + \hbar\, T_{f,f'}[h], \qquad T_{f,f'}[h] = \int f(x) f'(x')\, w^\mu[h](x) w^\nu[h](x') \partial_\mu^x \partial_\nu^{x'} \Delta_m^+(x - x')\, \mathrm{d}^4x\, \mathrm{d}^4x'.$$

It is the Wick contraction formula for the proposed $B_f \star_H B_{f'}$. The potential $\hbar^2$ coefficient vanishes.

Proof. For fixed smooth h, the factor multiplying the differentiated Hadamard distribution in is a smooth compact test function. Hence the pairing exists.

The map from h to $w[h]$, restricted to any fixed compact observation region, is continuous in the smooth topology and depends only on a fixed compact causal region. Polarization of therefore gives a continuous bilinear form on the corresponding smooth graviton configurations. The Schwartz kernel theorem gives a compact distributional quadratic kernel.

There are no h–φ contractions. Every term involving a w–w contraction vanishes by Proposition . This leaves exactly .

Remark. The conclusion is stronger than a merely formal inventory of self-contractions, but weaker than an algebra theorem. The mixed triple below shows why the distinction is necessary.

The mixed quadratic test

Let

$$Q_q = \int q^{\rho\sigma\nu}(y) h_{\rho\sigma}(y) \partial_\nu \varphi(y)\, \mathrm{d}^4 y, \qquad q \in C_c^\infty.$$

Proposition (The mixed quadratic test). *The finite contraction formulas for $B_f \star_H Q_q$ and $Q_q \star_H B_f$ define compact distributional polynomial kernels. In particular, the double contraction*

$$H^{\mu}_{wh\rho\sigma}(x,y)\, \partial^x_\mu \partial^y_\nu \Delta^+_m(x-y)$$

is a well-defined distribution.

Proof. Each component of H_{wh} has a direct and a reflected Fourier representation of the form illustrated in , with polynomial tensor numerators. In either branch, every nonzero wavefront covector in the *y*-slot has the same temporal orientation as the second slot of Δ^+_m; it is nonzero and null. This follows directly from the phase $irs - i\mathbf{p}\cdot\mathbf{y}$. The direct branch has the usual null relation and the reflected branch has the relation obtained by reflecting the first time coordinate.

The two factors in the displayed product therefore cannot have opposite total covectors: their *y*-components already have the same orientation in a pointed cone. The multiplication criterion applies.

A single *h*-contraction leaves a distribution multiplied by the smooth classical scalar factors. A single scalar contraction leaves a differentiated Hadamard distribution multiplied by the smooth factors $w[h](x)h(y)$. As in Proposition , continuity in the smooth configurations and the kernel theorem give the remaining compact polynomial kernels. The reverse product has the reversed, but again common, orientation.

The product with

$$W_g = \int g\, h_{00}^2$$

also exists at the level of $B_f \star_H W_g$: at most one graviton leg of B_f can contract with W_g, and the resulting mixed covariance is a distribution paired with smooth remaining configuration factors. What fails is the mandatory subsequent multiplication by a second insertion in the topology specified next.

The named topology and regular approximations

Normal-topology stages

For a compact $K \subset M \times M$, write

$$\mathscr{E}'_{\Gamma_B}(K) = \{u \in \mathscr{E}'(M \times M) : \operatorname{supp} u \subset K,\ \mathrm{WF}(u) \subset \Gamma_B\},$$

componentwise in the finite-dimensional tensor indices. It carries the normal topology.

Explicitly, this topology is generated by strong distribution seminorms

$$p_{\mathscr{B}}(u) = \sup_{\psi \in \mathscr{B}} |\langle u, \psi \rangle|,$$

where $\mathscr{B}$ is a bounded set of test functions, and by

$$p_{\chi, V, N}(u) = \sup_{\zeta \in V} (1 + |\zeta|)^N |\widehat{\chi u}(\zeta)|,$$

where V is a closed cone and

$$(\operatorname{supp} \chi \times V) \cap \Gamma_B = \varnothing.$$

The normal topology and the distinction between continuity and hypocontinuity are important here [1, 2].

The translation-invariant angular cone Λ_B in is deliberately an upper-bound cone. The theorem does not presume that it is the smallest possible cone.

A compact regularization of the kernels

Choose a real, nonnegative, even, Euclidean-radial $\rho \in C_c^\infty(\mathbb{R}^4)$ with $\int \rho = 1$, and put

$$\rho_\varepsilon(x) = \varepsilon^{-4} \rho(x/\varepsilon), \qquad S_\varepsilon u = \rho_{\mathrm{varepsilonu}}.$$

Define

$$B_{f,\varepsilon}[h, \varphi] = B_f[S_\varepsilon h, S_\varepsilon \varphi].$$

Its mixed kernel is

$$b_{f,\varepsilon} = (\rho_\varepsilon \otimes \rho_\varepsilon) * b_f.$$

In particular, $B_{f,\varepsilon} \in \mathcal{P}_{\mathrm{reg}}$.

These are approximation devices for the exact sharp kernel. They are not proposed as replacement observables.

Lemma (Normal convergence). *For every fixed f, choose a compact K containing a neighborhood of* $\operatorname{supp} b_f$. *Then, for all sufficiently small* $\varepsilon > 0$,

$$\operatorname{supp} b_{f,\varepsilon} \subset K, \qquad b_{f,\varepsilon} \to b_f \quad \text{in} \mathscr{E}'_{\Gamma_B}(K)$$

with its normal topology.

Proof. Smoothness and the fixed compact support follow from convolution with a compact approximate identity.

Strong distribution convergence follows from the finite order of b_f and the convergence

$$(\rho_\varepsilon \otimes \rho_\varepsilon) * \psi \to \psi$$

in all smooth seminorms, uniformly for ψ in a bounded set of test functions.

For completeness, the microlocal estimates can be checked directly because Γ_B has a translation-invariant angular cone. Compactness and $\mathrm{WF}(b_f) \subset (M \times M) \times \Lambda_B$ imply rapid decrease of $\widehat{b_f}$ in every closed cone separated from Λ_B. Moreover,

$$\widehat{b_{f,\varepsilon}}(\xi, \zeta) = \widehat{\rho}(\varepsilon\xi)\widehat{\rho}(\varepsilon\zeta)\widehat{b_f}(\xi, \zeta).$$

The multipliers are uniformly bounded and tend to one on compact frequency sets.

On a closed cone separated from Λ_B, split frequency space into a fixed ball and its complement. On the ball, the multiplier converges uniformly to one. On the complement, arbitrarily high rapid-decay bounds for $\widehat{b_f}$ give a uniformly small tail. Thus the weighted Fourier seminorms converge to zero.

For a localized seminorm, use

$$\widehat{\chi(b_{f,\varepsilon} - b_f)} = \hat{\chi}[(\hat{\rho}(\varepsilon\xi)\hat{\rho}(\varepsilon\zeta) - 1)\hat{b}_f].$$

A slightly larger cone still disjoint from Λ_B controls the nearby-frequency part. On the complementary part, the angular separation forces the argument of $\hat{\chi}$ to be large; its arbitrary rapid decrease dominates the uniform polynomial growth of $\hat{b}_f$. This proves convergence of every normal-topology seminorm.

Remark (Support and the role of the regulator). The approximants have support in arbitrarily small neighborhoods of the exact compact support; the limit retains the exact causal support of Proposition . The theorem uses an ordinary fixed-support distribution stage containing those neighborhoods. It does not impose an equation of motion and does not replace θ by a new smooth-edge observable.

The mandatory mixed triple

The exact regularized coefficient

Let $f, \dot{f}, g$ be real compact smooth functions, with $f, \dot{f}$ supported in M_+, and set

$$W_g[h] = \int g(y) h_{00}(y)^2 \, \mathrm{d}^4 y.$$

Define

$$\mathscr{G}_\varepsilon(f, g, \dot{f}) = \mathrm{ev}_0 [\hbar^3] \big(B_{f,\varepsilon} \star_H W_g \star_H B_{\dot{f},\varepsilon} \big).$$

This coefficient is unambiguously defined in the old polynomial domain.

There is exactly one fully contracted graph type: the scalar legs of the two insertions are joined, and the two graviton legs are joined to the two legs of W_g. The two assignments of the W_g legs give a factor of 2. No *ww* contraction is present in this graph.

Let

$$\mu(u) = \hat{\rho}(-u, u\mathbf{n}), \qquad \mathbf{n} \in S^2.$$

Euclidean radiality makes this independent of **n**. It is real, smooth, rapidly decreasing, and $\mu(0) = 1$. For the scalar field define

$$M_\varepsilon(r) = \hat{\rho}(\varepsilon(-\omega_m(r), r\mathbf{n})) = \mu\left(\varepsilon\sqrt{r^2 + \frac{m^2}{2}}\right).$$

Here and below $\omega_m(r) = \sqrt{r^2 + m^2}$.

Let K_ε be the right-hand side of with the additional multiplier $\mu(\varepsilon|\mathbf{p}|)$ in the momentum integral. This is exactly the covariance between $w[S_\varepsilon h](x)$ and $h_{00}(y)$.

For $q \in \mathbb{R}^3$, put

$$F_{f,\varepsilon}(y, q) = \int f(x) K_\varepsilon(x, y) e^{-i\omega_m(q)t + iq\cdot\mathbf{x}}\, \mathrm{d}^4x.$$

Lemma (Spectral form of the triple). *For every $\varepsilon > 0$,*

$$\mathscr{G}_\varepsilon(f, g, f') = \frac{1}{(2\pi)^3}\int g(y)\, \mathrm{d}^4y \int_{\mathbb{R}^3} \omega_m(q) M_\varepsilon(|q|)^2 F_{f,\varepsilon}(y, q)\overline{F_{f',\varepsilon}(y, q)}\, \mathrm{d}^3q.$$

For $f' = f$ and $g \geq 0$, this expression is nonnegative.

Proof. By , a contraction with h_{00} forces the insertion vector index to be 0. Thus the scalar contraction is

$$\partial_t\partial_{t'}(S_\varepsilon \otimes S_\varepsilon)\Delta_m^+(x - x') = \int \frac{\mathrm{d}^3q}{(2\pi)^3}\frac{\omega_m(q)}{2} M_\varepsilon(|q|)^2 e^{-i\omega_m(q)(t-t') + iq\cdot(\mathbf{x}-\mathbf{x}')}.$$

The factor 2 from the two W_g-leg assignments cancels the factor 1/2 in this spectral density. The two minus signs in the definitions of the insertions cancel as well. Hermiticity gives the complex conjugate factor in .

For fixed $\varepsilon > 0$, the mollifier multipliers and the compact smooth smearings make the resulting integrals convergent. The positivity assertion uses only the positive scalar spectral measure, not positivity of the graviton covariance.

The reflected light-ray transform

For $y = (s, \mathbf{y})$ and $\mathbf{n} \in S^2$, define

$$A_f(y,\mathbf{n}) = \int_{\mathbb{R}} f(t, \mathbf{y} + \mathbf{n}(t+s))\,\mathrm{d}t.$$

Only a compact interval contributes. Geometrically, this samples a future observation ray issuing from the spatial point $\mathbf{y} + s\mathbf{n}$ on Σ_0.

Lemma (Uniform high-frequency asymptotics). *Let $Y \subset M$ be compact. Uniformly for $y \in Y$, $\mathbf{n} \in S^2$, $r \geq 1$, and $0 < \varepsilon \leq 1$,*

$$F_{f,\varepsilon}(y, r\mathbf{n}) = -\frac{i}{8r^2} e^{ir(s + \mathbf{n}\cdot\mathbf{y})}\big[\mu(\varepsilon r) A_f(y,\mathbf{n}) + E_{f,\varepsilon}(y,r,\mathbf{n})\big],$$

where

$$|E_{f,\varepsilon}(y,r,\mathbf{n})| \leq C_{f,Y,m}(r^{-1} + \varepsilon).$$

For $|q| \leq 1$, $F_{f,\varepsilon}(y,q)$ is uniformly bounded on Y, independently of $0 < \varepsilon \leq 1$.

Proof. Separate into its direct and reflected terms.

Direct term. After integration against f, the direct term contains the Fourier transform of f, or of tf, at temporal frequency $|p| + \omega_m(r)$ and spatial frequency $p + r\mathbf{n}$. Since f is smooth and compact,

$$\left|\widehat{f}(|p| + \omega_m(r), p + r\mathbf{n})\right| \leq C_N(1 + |p| + r)^{-N},$$

and the same holds with tf. Integrating against $|p|^{-1} + |p|^{-2}$ proves arbitrary rapid decrease in r, uniformly in ε and in $y \in Y$.

Reflected term. Write

$$p = -r\mathbf{n} + a.$$

On $|a| \leq r/2$,

$$|p| = r - \mathbf{n}\cdot a + O(|a|^2/r),$$

$$r^2|p|^{-2} = 1 + O(|a|/r),$$

$$\mu(\varepsilon|p|) = \mu(\varepsilon r) + O(\varepsilon|a|),$$

$$\omega_m(r) = r + O(r^{-1}).$$

The constants in the last estimate may depend on the fixed mass. The phase is

$$|p|(t+s) - \omega_m(r)t + p\cdot(\mathbf{x}-\mathbf{y}) + r\mathbf{n}\cdot\mathbf{x}$$

$$= r(s + \mathbf{n}\cdot\mathbf{y}) + a\cdot(\mathbf{x}-\mathbf{y}-\mathbf{n}(t+s)) + O\big((|a|^2+1)/r\big),$$

uniformly for t in the support of f and $y \in Y$.

The spatial Fourier transform of $f(t,\cdot)$ decreases rapidly in a, uniformly in t, with all required derivatives. The errors above therefore contribute $O(r^{-1}+\varepsilon)$ after extracting r^{-2}.

The leading a-integral is Fourier inversion:

$$\int_{\mathbb{R}^3} \frac{\mathrm{d}^3 a}{(2\pi)^3} \int_{\mathbb{R}^3} f(t,\mathbf{x}) e^{ia\cdot(\mathbf{x}-\mathbf{y}-\mathbf{n}(t+s))}\,\mathrm{d}^3x = f(t,\mathbf{y}+\mathbf{n}(t+s)).$$

Its t-integral is A_f.

On $|a| > r/2$, arbitrary rapid decrease in a dominates the locally integrable factor $|p|^{-2}$. After extracting r^{-2}, that tail is $O(r^{-N})$ for any prescribed N, by taking sufficiently many Fourier-decay powers. The direct term can be absorbed into the same error. This proves –.

For bounded q, integrate the spatial Fourier-decay bounds against $|p|^{-1} + |p|^{-2}$. Both are locally integrable, and the large-$|p|$ tails are rapidly decreasing. This proves the last assertion.

The logarithmic coefficient

Theorem (Explicit mixed-graph divergence). *For real $f, \dot{f}, g$ as above,*

$$\mathscr{G}_\varepsilon(f,g,\dot{f}) = \mathscr{C}(f,g,\dot{f})\log(\varepsilon^{-1}) + O(1),$$

where

$$\text{boxed } \mathscr{C}(f,g,f') = \frac{1}{64(2\pi)^3}\int_M g(y)\,\mathrm{d}^4y \int_{S^2} A_f(y,\mathbf{n})A_{f'}(y,\mathbf{n})\,\mathrm{d}\Omega(\mathbf{n}).$$

The $O(1)$ bound is for the fixed three test functions and fixed mass. The coefficient of the logarithm is independent of the chosen normalized radial mollifier.

Proof. Insert Lemma into and use polar coordinates $q = r\mathbf{n}$. For $r \geq 1$,

$$r^2\omega_m(r) = r^3(1 + O(r^{-2})).$$

The leading contribution is therefore

$$\mathscr{C}(f,g,f')\,I_\varepsilon, \qquad I_\varepsilon = \int_1^\infty \frac{M_\varepsilon(r)^2\mu(\varepsilon r)^2}{r}\,\mathrm{d}r.$$

We first check that the remainder is uniformly bounded. By , the error in the product of the two square brackets in is bounded by a constant times

$$(r^{-1} + \varepsilon) + (r^{-1} + \varepsilon)^2.$$

Also, for every N,

$$|M_\varepsilon(r)| \leq C_N(1 + \varepsilon r)^{-N}.$$

Thus the error integral is controlled by

$$C\int_1^\infty \frac{(1 + \varepsilon r)^{-N}}{r}\left[r^{-1} + \varepsilon + (r^{-1} + \varepsilon)^2\right]\mathrm{d}r.$$

This is uniformly bounded: terms containing r^{-1} are integrable without the cutoff, and

$$\varepsilon\int_1^\infty \frac{(1 + \varepsilon r)^{-N}}{r}\,\mathrm{d}r = O(\varepsilon\log(\varepsilon^{-1}) + \varepsilon).$$

The correction from $\omega_m(r)/r - 1 = O(r^{-2})$ is integrable as well. The region $0 \leq r \leq 1$ is uniformly bounded by the final assertion of Lemma .

It remains to evaluate I_ε. For $r \geq 1$,

$$\left|\sqrt{r^2+\frac{m^2}{2}}-r\right| \le C_m r^{-1},$$

so boundedness of μ' gives

$$|M_\varepsilon(r)-\mu(\varepsilon r)| \le C_m \varepsilon r^{-1}.$$

Consequently,

$$I_\varepsilon = \int_1^\infty \frac{\mu(\varepsilon r)^4}{r}\,\mathrm{d}r + O(\varepsilon).$$

Changing variables $u = \varepsilon r$,

$$\int_1^\infty \frac{\mu(\varepsilon r)^4}{r}\,\mathrm{d}r = \int_\varepsilon^\infty \frac{\mu(u)^4}{u}\,\mathrm{d}u = \log(\varepsilon^{-1}) + O(1),$$

because $\mu(0) = 1$, $\mu(u)^4 - 1 = O(u)$ near zero, and μ decreases rapidly at infinity. This proves with the coefficient .

Nonzero coefficient away from all observation coincidences

Lemma (Separated-support witness). *There exist nonnegative $f, f', g \in C_c^\infty(M_+)$ with pairwise disjoint supports and*

$$\mathscr{C}(f,g,f') > 0.$$

Their supports may also be chosen disjoint from Σ_0.

Proof. Choose $S, T, T' > 0$ pairwise distinct and $\mathbf{n}_0 \in S^2$. Consider

$$y_0 = (S,0), \qquad x_0 = (T,(T+S)\mathbf{n}_0), \qquad x_0' = (T',(T'+S)\mathbf{n}_0).$$

These three spacetime points are distinct. Choose nonnegative smooth bumps g, f, f', respectively positive near these points, with sufficiently small pairwise disjoint supports contained in M_+.

For y near y_0 and $\mathbf{n}$ near $\mathbf{n}_0$, the rays in meet the regions where f and f' are positive. Hence both ray transforms are positive on an open set on which g is positive. Formula then gives the result.

The coefficient $\mathscr{C}$ is supported on the reflected collinearity relation

$$\begin{aligned}\mathscr{R}_{\Sigma} = \{(x, y, x') : \quad & y = (s, \mathbf{y}), \\ & x = (t, \mathbf{y} + \mathbf{n}(t + s)), \\ & x' = (t', \mathbf{y} + \mathbf{n}(t' + s)), ?\mathbf{n} \in S^2\}.\end{aligned}$$

The witness in Lemma lies on its noncoincident part.

The continuity obstruction

The coefficient-level statement

For regular mixed kernels u, v, let F_u, F_v denote the associated degree-$(1,1)$ polynomials, and define

$$T_g(u, v) = \mathrm{ev}_0[\hbar^3](F_u \star_H W_g \star_H F_v).$$

This is a bilinear map on regular kernels.

Theorem (No normal-topology extension of the mandatory coefficient). *Choose f, f', g as in Lemma , and let K be a compact set containing neighborhoods of both mixed-kernel supports. The map T_g has no extension to a domain containing*

$$b_f,\ b_{f'}, ?\{b_{f,\varepsilon}, b_{f',\varepsilon} : 0 < \varepsilon \le \varepsilon_0\}$$

which:

1. *agrees with T_g on the regular kernels;*

2. *is sequentially continuous at $(b_f, b_{f'})$ for the product of the subspace normal topologies inherited from $\mathscr{E}'_{\Gamma_B}(K)$.*

Proof. By Lemma ,

$$(b_{f,\varepsilon}, b_{f',\varepsilon}) \to (b_f, b_{f'})$$

in the stated product topology. Sequential continuity would force $T_g(b_{f,\varepsilon}, b_{f',\varepsilon})$ to converge to a finite complex number.

Agreement on regular kernels and Theorem instead give

$$\operatorname{Re} T_g(b_{f,\varepsilon}, b_{f',\varepsilon}) = \mathscr{C}(f, g, f') \log(\varepsilon^{-1}) + O(1) \to +\infty.$$

This is a contradiction.

Remark (Why this is more than a divergent regulator). Three independently established facts enter the contradiction: the approximants belong to the old domain; they converge to the sharp kernels in the named topology; and the proposed extension is required to agree with the old products and be continuous. The divergence alone would not constitute an obstruction.

Consequence for algebra extensions

Assumption (Normal-topology Wick extension). A proposed extension consists of a Hausdorff locally convex $\mathbb{C}[[\hbar]]$-algebra $\mathscr{A}$, with the following properties.

1. $\mathcal{P}_{\mathrm{mc}}[[\hbar]]$ embeds into $\mathscr{A}$, and its product is the fixed Wick product .

2. The algebra admits the sharp insertions $Q_\Sigma(B_f)$, with

$$[Q_\Sigma(B_f)]_{\hbar^0} = B_f$$

as functionals on all smooth configurations.

3. On the degree-$(1,1)$ kernel stages used above, admission is sequentially continuous for the normal topology and agrees with the old inclusion on regular kernels. In particular,

$$B_{f,\varepsilon} \to Q_\Sigma(B_f) \quad \text{in } \mathscr{A}.$$

4. Multiplication is hypocontinuous with respect to bounded sets. The field-independent $\hbar^3$ coefficient map

$$a \text{longmapsto} \operatorname{ev}_0[\hbar^3]a$$

is continuous and agrees with the usual coefficient extraction on the old polynomial domain.

Corollary (No algebra extension under the stated requirements). *There is no extension satisfying Assumption .*

Proof. Let

$$X_\varepsilon = B_{f,\varepsilon}, \quad Y_\varepsilon = B_{f',\varepsilon}, \quad X = Q_\Sigma(B_f), \quad Y = Q_\Sigma(B_{f'}).$$

By assumption $X_\varepsilon \to X$ and $Y_\varepsilon \to Y$, so both convergent families are bounded.

Hypocontinuity implies separate continuity, hence $X_\varepsilon \star_H W_g \to X \star_H W_g$. It also gives joint sequential continuity along convergent sequences: subtract the limit product, split the difference into two terms, and use continuity uniformly over the bounded family in the other factor. Therefore

$$(X_\varepsilon \star_H W_g) \star_H Y_\varepsilon \to (X \star_H W_g) \star_H Y.$$

Applying the continuous coefficient map gives convergence of $\mathscr{G}_\varepsilon(f, g, f')$, contradicting Theorem and Lemma .

Theorem follows from Lemma , Theorem , Lemma , and Theorem .

Counterterms and the scope of the conclusion

The extension rule considered here is: Keep the fixed Wick product on the old domain, admit sharp kernels continuously in the specified normal topology, and introduce no new counterterms on noncoincident characteristic relations.

The separated-support witness shows that a counterterm supported only on the ordinary observation diagonals

$$x = y, \qquad y = x', \qquad x = x'$$

or on observation boundary faces cannot subtract the coefficient . The divergence persists when the three observation supports avoid all those sets.

There is an additional distinction worth making explicit. A distribution local in auxiliary Cauchy variables can acquire nonlocal support after Cauchy propagation. Such a modification is not automatically an ordinary observation-diagonal counterterm. Its pushforward, topology, and compatibility with the old Wick product would have to be specified separately.

More fundamentally, any subtraction performed on the regular approximants in changes products that are already fixed in $\mathcal{P}_{\mathrm{mc}}$. Assigning a subtracted value only at the sharp kernel does not restore the continuity contradicted by

Theorem .

Thus the theorem excludes the stated normal-topology extension. It does not exclude a different boundary theory with additional renormalized operations, a stronger topology in which the approximants do not converge, or a smaller bulk algebra that omits W_g.

Outcome and the remaining interacting problem

Audit of the finite tests

toprule Test Object Result in this manuscript

midrule A Smoothly smeared Cauchy and bulk linear fields Cauchy covariance construction exists.

B B_f against bulk linear fields Compact polynomial representatives exist.

C $B_f \star_H B_{f'}$ The kernel exists; the *ww* terms vanish.

D $B_f \star_H Q_q$ and reverse order The finite contraction kernels exist.

E $B_f \star_H W_g \star_H B_{f'}$ No extension of the fully contracted coefficient with the stated normal-topology continuity and old-product agreement.

bottomrule

Neither the vanishing of H_{ww} nor success of Tests A–D is a closure theorem. Test E supplies the named obstruction.

The characteristic-cone alternative also fails for the actual insertion kernel by Proposition . Merely allowing timelike linear boundary conormals misses the same-orientation null pairs in the bilinear kernel.

Off-shell normalization and presentation independence

Throughout the argument:

• B_f is exactly the functional ;

• j = square h is retained in ;

• no free-equation quotient is taken;

• no independent coordinate field is introduced;

• all regularized coefficients are computed in the original Wick-symbol presentation.

The bulk representation , the Cauchy-data/source representation , and the Fourier expression describe the same fixed operator. The obstruction is therefore not obtained by assigning different products to two presentations of the same classical kernel.

Separation from time-ordered and retarded products

The source-sector observation in Section 2 uses the bisolution identity

$$PH = 0.$$

A Feynman Green function instead satisfies

$$PH_F = c\,\delta$$

with a convention-dependent nonzero constant c. Consequently, source variables with zero free Wick contractions can produce contact terms in time-ordered constructions. The free Wick and renormalized time-ordered constructions are distinct [4].

No statement here constructs any of

$$R_1(V_1^{\mathrm{sc}}; B_f), \qquad \mathscr{B}_f^{(2)}, \qquad R_1(V_2; \mathscr{B}_f^{(0)}), \qquad \frac{1}{2}R_2(V_1, V_1; \mathscr{B}_f^{(0)}).$$

No interaction vertex is needed for the obstruction, and none is introduced as an unused premise.

Closed statement

The result of this manuscript is the following restricted but complete conclusion:

$$\text{boxed}\begin{array}{c}\text{sharp zero-Cauchy dressing + exact off-shell insertion}\\ +\text{the fixed de Donder Wick matrix + normal-topology admission}\\ \text{+continuity and agreement with the old Wick product}\\ \Rightarrow \text{failure of the mandatory mixed triple coefficient.}\end{array}$$

This is a C-obstruction for a named boundary-extension scheme. It is not a proof that every possible enlargement fails, not a bimodule construction, and not a closure of the interacting product problem.

appendix

A compact derivation of the boundary-frequency reversal

The mechanism can be isolated at one massless momentum. For $r > 0$, the scalar zero-Cauchy inverse is

$$\mathrm{E}_0(e^{-irt}) = -\frac{it}{2r}e^{-irt} + \frac{1}{4r^2}(e^{irt} - e^{-irt}).$$

The second term enforces the initial conditions. It introduces an e^{irt} component although the source has only e^{-irt} frequency.

For the component selected by h_{00}, the derivative in multiplies the source by $-ir$, and the Hadamard spectral measure contributes $1/(2r)$. The reflected mixed covariance therefore has amplitude

$$-\frac{i}{8r^2}.$$

In the mixed triple, two such amplitudes contribute r^{-4}. The differentiated scalar covariance contributes r, and the remaining radial measure contributes $r^2\,\mathrm{d}r$. Thus the leading radial behavior is

$$\frac{\mathrm{d}r}{r}.$$

This power count alone is not the proof of divergence: the tensor selection, exact graph identity, nonzero light-ray coefficient, uniform remainder estimate, and topology argument are supplied in Sections 4–7.

Why no additional boundary field is hidden in the proof

The variables

$$(\gamma_0 h, \gamma_1 h, \square h)$$

are used only through the identity

$$h = \mathrm{U}(\gamma_0 h, \gamma_1 h) + \mathrm{E}_0 \square h.$$

Their compatibility is inherited from the original smooth configuration h. No independent integration over boundary data or source variables is introduced.

Similarly, $S_\varepsilon h$ and $S_\varepsilon \varphi$ in are ordinary smoothings of the original fields. The resulting $B_{f,\varepsilon}$ are regular elements of the old domain and serve only to test continuity at the exact sharp kernel. The proof does not declare any fixed $\varepsilon > 0$ observable to be the sharp insertion.

Book expansion notes (t433u)

The mixed-Wick algebra remains unconstructed. VII-D [11] supplies the finite part of the same regulated family and is not a repair of this obstruction. Observation-boundary source record: [12].

Standing ledger. Not spacetime emergence; not Einstein dynamics; not a Page curve, island, bounce, vacuum-energy selection, or de Sitter state as an Enter theorem; not a derived metric. O.1 stays NOT_CLOSED. XVI-N is not discharged. XV-B is not opened. No quantum coefficient is admitted. Host C is not created. Dom (T) remains undetermined; J2 remains unresolved. Problems 6, 7, and 9 are named, not solved. Gate 0 and ST4b stay NOT_SUPPLIED. Host-A stays NAMED_COMPARISON_ONLY $\{\mathrm{GI}, \mathrm{CB}\}$. The shredder is not Gibbons–Hawking. Cited sheets are source records, not premises. Conjectures stay conjectures. wired=false.

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