A particle-supported slicing event on a fixed spinless cone

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Abstract

We construct an explicit one-parameter family of embedded spacelike polyhedral slices of a fixed spinless conical spacetime with deficit $\mu = \pi/2$. The family agrees with a prescribed resting exterior and has exactly one incidence event: the link of the particle changes from three incident faces to six. Six vertices, twelve edges, and six faces are born at the particle; their coordinates, lengths, areas, and one-sided laws are given exactly. A finite table of adapted affine charts yields identity products around regular vertices and the full spinless conical identification around a primitive meridian. The incoming germ, outgoing germ, and annular collar have the same framed affine peripheral holonomy. For an explicitly stated geometry-only interface, the event has spectator-blind control, a forward-domain decoder, and an exact NOT–CNOT–Toffoli implementation with separately reserved location and history storage. The result is a kinematic consistency certificate for one prescribed slicing event.

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Language en

Problem, interface, and scope

The object of this paper is one incidence event involving the actual singular worldline of a fixed conical spacetime. The event is not obtained by attaching a particle label to a vacuum exchange, and its support is not enlarged merely to include an otherwise unused particle. The particle's combinatorial link changes from a three-cycle to a six-cycle.

The conical description of a stationary spinless particle is classical [1]. Particle-associated polygon transitions also occur in the classical polygon literature [2, 3]. We do not claim to introduce particle-supported transitions as such. The contribution here is an explicit construction together with a complete geometric and finite-interface certificate. Polygon and holonomy terminology is compatible with the language of [4]; no constraint-algebra result from that work is used.

All mathematical assumptions needed below are stated explicitly. In particular, the construction does not depend on an unpublished length table or an unspecified reversible compiler.

Interface contract (Geometry-only interface). A configuration contains:

1. a labelled polygonal slice or a labelled one-sided slice germ;

2. a geometric particle packet specifying an identifier, deficit, worldline, reference rest vector, and particle-cell label;

3. an exact clock and a finite permission register;

4. for a fixed finite list of original matter names, one declared cell location per name;

5. an internal matter state. A geometric rule, called a *G*-rule, may inspect the declared geometric data and

geometric control registers. Its guard, geometric output, certificates, and cell-label transport must not depend on the number of matter names or on their internal state. Applying a fixed cell-label transport to each original name is permitted.

At the event considered here:

1. no measurement or coupling rule is applied;

2. the internal matter factor is acted on by the identity;

3. the particle identifier is geometric data, not a test of matter occupancy;

4. rounded reports are auxiliary and do not determine the event guard;

5. implementation history is not a geometric observation;

6. decoder correctness is required on a declared forward event domain, not on intermediate compiler states.

The three-way distinction between geometric, measurement, and coupling rules is retained. The measurement primitive Enter is not used.

Definition (Admitted polygonal objects in this paper). Faces are embedded flat polygonal disks in spacelike affine planes. They need not be convex. The fixed exterior may contain unbounded polygonal faces. Regular vertices need not be trivalent.

At a singular event parameter, collapsed cells are not positive dimensional cells of the slice. Their labels and one-sided formulas may be retained as *germ metadata*. A change between two such labelled germs is an admissible event when the two germs have the same underlying limiting slice and the prescribed common exterior.

Remark (Exact scope of the interface conclusion). The interface theorem below concerns Contract and Definition . It is not an admission theorem for a separate calculus that imposes additional restrictions, such as trivalent regular vertices, elementary four-face exchanges, convex faces only, or equal proper-time increments in every face frame. Compatibility with such an additional restriction would require a separate check.

Thus the geometric existence theorem is unconditional for the objects defined here; its use in any frozen external framework is conditional only on that framework admitting these explicitly described germs. No unprovided admission predicate is silently enlarged.

What changes at the event

The construction is a *three-to-six particle-link expansion*. For negative parameters, three sloping faces meet at the particle. For positive parameters, six different sloping faces meet there. The old three faces remain in the cellulation, but cease to be incident to the particle.

This is one isolated event in the prescribed one-parameter family. It is a simultaneous local expansion, not a claim of genericity in the space of all slicings and not an identification with one particular elementary transition from a classical catalogue.

The marked cone and a restricted obstruction

Ambient spacetime

Use Minkowski coordinates $z = (T, X, Y)$ and metric

$$\eta = \operatorname{diag}(-1, 1, 1).$$

Fix

$$\mu = \frac{\pi}{2}, \qquad \beta = 1 - \frac{\mu}{2\pi} = \frac{3}{4}, \qquad L = 2\pi\beta = \frac{3\pi}{2}.$$

The spatial cone C_μ is obtained from

$$\{(r, \theta) : r \geq 0,\ 0 \leq \theta \leq L\}$$

by identifying $(r, 0)$ with (r, L) and identifying all points with $r = 0$. Its metric away from the apex is

$$dr^2 + r^2 d\theta^2.$$

The fixed spacetime is

$$M = \mathbb{R}_T \times C_\mu, \qquad ds_M^2 = -dT^2 + dr^2 + r^2 d\theta^2.$$

Its singular worldline is

$$\mathscr{W} = \{(T, 0) : T \in \mathbb{R}\}.$$

In the reference development, $\mathscr{W}$ is the line $\{(T, 0, 0)\}$.

For later use set

$$v_i = \begin{pmatrix} \cos(i\pi/2) \\ \sin(i\pi/2) \end{pmatrix}, \qquad i = 0, 1, 2, 3.$$

The cone consists of three developed quadrants

$$S_i = \{\xi v_i + \eta v_{i+1} : \xi, \eta \geq 0\}, \qquad i = 0, 1, 2.$$

The ray in direction v_3 is identified with the ray in direction v_0. Indices on cone rays and cell labels are taken modulo three; the developed vectors $v_0, \ldots, v_3$ are kept distinct.

The upper seam is mapped to the lower seam by

$$J = (R(\mu), 0), \qquad R(\mu) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Thus $J(T, 0, -r) = (T, r, 0)$, and J fixes $\mathscr{W}$ pointwise.

Remark (Wedge gluing is not a finite rotation quotient). The cone is defined by gluing the two boundary rays of the $3\pi/2$ wedge. It is not the quotient of the entire Minkowski plane by the finite group generated by $R(\pi/2)$. The latter construction would have a different opening angle.

Holonomy convention

A positive meridian increases the lifted angular coordinate by L. Its holonomy will mean the ordered product of affine coordinate transitions encountered along the positively oriented loop, with the last transition acting on the left.

With this convention, crossing the upper seam into the lower seam contributes J. The increasing-angle developing map on the universal cover satisfies the inverse equivariance relation

$$\mathrm{dev}\,(r, \theta + L) = R(-\mu)\mathrm{dev}\,(r, \theta).$$

There is no sign contradiction: deck equivariance and the ordered coordinate-transition product use inverse conventions.

The affine reference coordinates retain their origin on $\mathscr{W}$. A basepoint for a loop does not mean that the affine coordinates are re-centred at that basepoint.

The literal-transplant obstruction

Lemma (Obstruction for the axis-aligned vacuum maximum). *Consider*

$$F_s^{\mathrm{vac}}(x, y) = \max\{p|x| + as,\ q|y| + bs\}, \quad a = \sqrt{1 - p^2}, \quad b = \sqrt{1 - q^2},$$

on the axis-aligned wedge with seam $(r, 0) \sim (0, -r)$. *If* $p \neq q$, *this function does not descend continuously to the cone at* $s = 0$.

In particular, the choice $p = 3/5$, $q = 5/13$ *does not define a particle-supported slice on this seam. A positive upper cap does not repair the defect near the apex. If* $p = q$, *the displayed family has no rate-driven incidence event.*

Proof. For every $r > 0$,

$$F_0^{\mathrm{vac}}(r, 0) = pr, \qquad F_0^{\mathrm{vac}}(0, -r) = qr.$$

These values must agree at identified points. They do not agree when $p \neq q$.

A fixed positive upper cap is inactive for sufficiently small r, so the same mismatch persists near the apex. If $p = q$, then $a = b$ and

$$F_s^{\mathrm{vac}}(x, y) = as + p\max\{|x|, |y|\}.$$

The graph changes only by time translation; its incidence structure is independent of s.

This is a restriction on one named ansatz and one seam. It is not an obstruction to other slicing families.

An explicit seam-compatible slicing family

Three single-valued functions on the cone

In each sector S_i, with coordinates (ξ, η), define

$$\rho = \xi + \eta, \qquad h = \frac{2}{3}(\xi + \eta), \qquad k = \frac{3}{5}\min\{\xi, \eta\}.$$

These definitions agree on every common ray. In particular, on each ray,

$$\rho = r, \qquad h = \frac{2}{3}r, \qquad k = 0.$$

They therefore define continuous single-valued functions on C_μ.

Fix the parameter window $-1 \le s \le 1$ and set

$$\text{boxed}\, F_s(x) = s + \min\{2,\ \max\{h(x), s + k(x)\}\}.$$

The slices are

$$\Sigma_s = \{(F_s(x), x) : x \in C_\mu\}.$$

The constants 2/3 and 3/5 are chosen for this construction. They are not inherited from the obstructed vacuum maximum.

Lemma (Location of the active region). *For all $x \in C_\mu$,*

$$h(x) - k(x) \ge \frac{11}{30}\rho(x).$$

Consequently:

1. *if $s \le 0$, then*

$$F_s = s + \min\{2, h\};$$

2. *if* $s > 0$*, the branch* $s + k$ *can dominate* h *only where*

$$\rho \le \frac{30s}{11} \le \frac{30}{11} < 3;$$

3. *for all* $s \in [-1, 1]$ *and all* $\rho \ge 3$*,*

$$F_s = 2 + s.$$

Proof. Since $\min\{\xi, \eta\} \le (\xi + \eta)/2$,

$$h - k \ge \left(\frac{2}{3} - \frac{3}{10}\right)\rho = \frac{11}{30}\rho.$$

This proves the first two assertions. For $\rho \ge 3$, $h \ge 2$, so the outer minimum in equals 2.

Spacelikeness, ordering, and embedding

We use the following elementary graph criterion.

Lemma (Polyhedral graph criterion). *Let* F_s *be continuous functions on a Euclidean cone, piecewise affine in finitely many developed sectors. Suppose that every affine branch has spatial gradient of norm at most* $\lambda < 1$*. Then the graph of* F_s *is embedded and spacelike on each face, and*

$$|F_s(x) - F_s(y)| \le \lambda\, d_C(x, y).$$

If, in addition,

$$F_{s_2}(x) - F_{s_1}(x) \ge c(s_2 - s_1)$$

for $s_2 > s_1$ *and* $c > 0$*, then the graphs are strictly time ordered. If* $F_s = H + s$ *near the boundary of a compact spatial patch, they match the prescribed translating resting exterior there.*

Proof. On a face with gradient a, the induced metric is

$$I - aa^{\mathrm{T}}.$$

For every spatial tangent vector v,

$$v^{\mathrm{T}}(I - aa^{\mathrm{T}})v \geq (1 - \lambda^2)|v|^2 > 0.$$

Integrating the gradient bound along piecewise linear spatial paths and taking the infimum of path lengths gives the global Lipschitz bound. The projection of a graph to the cone is a homeomorphism, so the graph is embedded.

In the ultrastatic product, a causal curve between two points requires its time difference to be at least the spatial distance. The strict Lipschitz bound therefore also gives acausality between distinct graph points. At the apex, spacelikeness is understood in this uniformly Lipschitz, stratified sense.

The ordering and exterior statements follow directly from their respective hypotheses.

Proposition (The family is embedded, spacelike, and ordered). *The functions descend through the seam and define embedded spacelike slices. Uniform constants are*

$$\lambda = \frac{2\sqrt{2}}{3} < 1, \qquad 1 - \lambda^2 = \frac{1}{9}, \qquad c = 1.$$

More precisely, for $s_2 > s_1$,

$$s_2 - s_1 \leq F_{s_2}(x) - F_{s_1}(x) \leq 2(s_2 - s_1)$$

at every spatial point.

Proof. Single-valuedness was established before . The nonconstant gradients are

$$\left(\frac{2}{3}, \frac{2}{3}\right), \qquad \left(\frac{3}{5}, 0\right), \qquad \left(0, \frac{3}{5}\right),$$

up to sector rotations. Their norms are bounded by $2\sqrt{2}/3$.

For fixed x, the function

$$s \operatorname{longmapsto} \min\{2, \max\{h(x), s + k(x)\}\}$$

is nondecreasing and 1-Lipschitz. Adding s gives the displayed two-sided ordering bound. Lemma applies.

Support and collar

Let

$$K = \{x : \rho(x) \le 4\}, \qquad K_s = \{(F_s(x), x) : x \in K\}.$$

The moving core is $\{\rho < 3\}$. Choose the collar

$$\mathscr{C}_s = \{(2 + s, x) : 7/2 \le \rho(x) \le 4\}.$$

It is an annulus, disjoint from $\mathscr{W}$, and agrees with the fixed resting exterior translated in time by s.

Here and below, ``fixed exterior'' means fixed spatial cellulation, fixed transition data, and the prescribed equation $T = 2 + s$. It cannot mean an identical embedded subset of spacetime for every s, since the slices are strictly ordered.

The particle intersection is

$$P_s = \Sigma_s \cap \mathscr{W} = \begin{cases} (s, 0), & s \le 0, \\ (2s, 0), & s \ge 0. \end{cases}$$

Thus $P_s \in K_s$ for the entire window, including $s = 0$. The worldline itself has not changed.

Cellulation and the isolated particle event

Vertices and faces

Write $A_i = 3v_i$ and $D_i = 4v_i$ for spatial vertices on the cone. Their lifted time coordinate is $2 + s$.

For $s > 0$, set

$$a_s = \frac{3}{2}s, \qquad u_s = \frac{15}{11}s,$$

and introduce

$$B_i = a_s v_i, \qquad C_i = u_s(v_i + v_{i+1}).$$

All coordinates in this section are spatial unless a time coordinate is explicitly supplied.

The complete vertex data in one sector are

$c\|c\|c$vertex	sector coordinates	T
hline P	$(0,0)$	s $(s \le 0)$, $2s$ $(s \ge 0)$
A_i	$(3,0)$	$s+2$
A_{i+1}	$(0,3)$	$s+2$
D_i	$(4,0)$	$s+2$
D_{i+1}	$(0,4)$	$s+2$
B_i	$(\frac{3}{2}s,0)$	$2s$
B_{i+1}	$(0,\frac{3}{2}s)$	$2s$
C_i	$(\frac{15}{11}s,\frac{15}{11}s)$	$\frac{31}{11}s$

The last three rows are regular vertices only for $s > 0$.

For $s < 0$, the core face in sector i is the triangle

$$V_i = [P, A_i, A_{i+1}], \qquad T = s + \frac{2}{3}(\xi + \eta).$$

For $s > 0$, replace it by the following three faces:

$$U_i^- = [P, B_i, C_i], \qquad T = 2s + \frac{3}{5}\eta,$$

$$U_i^+ = [P, C_i, B_{i+1}], \qquad T = 2s + \frac{3}{5}\xi,$$

$$V_i = [B_i, A_i, A_{i+1}, B_{i+1}, C_i], \; T = s + \frac{2}{3}(\xi + \eta).$$

The face V_i is a simple, generally nonconvex pentagon.

For every s, the collar-side face is

$$Q_i = [A_i, D_i, D_{i+1}, A_{i+1}], \qquad T = s + 2.$$

Beyond $\rho = 4$, retain the fixed unbounded face

$$Z_i = \{(\xi, \eta) \in S_i : \xi + \eta \ge 4\}, \qquad T = s + 2.$$

The three exterior rays starting at the D_i are also retained.

Lemma (Exact branch decomposition). *The faces just listed form the graph of F_s without overlap of face*

interiors or omitted regions.

Proof. For $s \le 0$, Lemma gives the asserted three core triangles and resting exterior.

For $s > 0$, in the half-sector $\xi \ge \eta$,

$$h - k = \frac{2}{3}\xi + \frac{1}{15}\eta.$$

Hence the branch boundary is

$$10\xi + \eta = 15s.$$

It joins

$$B_i = \left(\frac{3}{2}s, 0\right) \quad \text{to} \quad C_i = \left(\frac{15}{11}s, \frac{15}{11}s\right).$$

In the other half-sector the boundary is

$$\xi + 10\eta = 15s,$$

joining C_i to B_{i+1}.

The region where $s + k$ dominates is precisely the union of the two triangles $[P, B_i, C_i]$ and $[P, C_i, B_{i+1}]$. The remainder of the sector triangle $\rho \le 3$ is the stated pentagon V_i. Since

$$\rho(C_i) = \frac{30s}{11} \le \frac{30}{11} < 3,$$

the inner boundary is strictly separated from the outer crest. The cap is active exactly in the prescribed exterior.

Edges, lengths, and areas

Use the following persistent edge labels:

$$e_i = \begin{cases} [P, A_i], & s < 0, \\ [B_i, A_i], & s > 0, \end{cases} \quad c_i = [A_i, A_{i+1}], \quad t_i = [A_i, D_i], \quad o_i = [D_i, D_{i+1}].$$

The new edge labels for $s > 0$ are

$$u_i = [P, B_i], \qquad v_i^{\mathrm{edge}} = [P, C_i], \qquad w_i^{-} = [B_i, C_i], \qquad w_i^{+} = [C_i, B_{i+1}].$$

The superscript on v_i^{edge} distinguishes an edge label from the developed direction vector v_i.

Proposition (Complete finite-edge length table). *The induced lengths of all finite edges in K_s are:*

$c\|c\|c\|$edge type	multiplicity	$s < 0$	$s > 0$
hline e_i	3	$\sqrt{5}$	$\sqrt{5}(1 - s/2)$
c_i	3	$3\sqrt{2}$	$3\sqrt{2}$
t_i	3	1	1
o_i	3	$4\sqrt{2}$	$4\sqrt{2}$
u_i	3	inactive	$\frac{3}{2}s$
v_i^{edge}	3	inactive	$\frac{3\sqrt{41}}{11}s$
w_i^{-}	3	inactive	$\frac{3\sqrt{65}}{22}s$
w_i^{+}	3	inactive	$\frac{3\sqrt{65}}{22}s$

Every active edge has positive length on its open parameter interval. Every new edge vanishes linearly at $s = 0$.

Proof. For an edge in a developed chart,

$$\ell^2 = |\Delta x|^2 - (\Delta T)^2.$$

Along e_i, the spatial displacement is $3v_i$ before the event and $(3 - \frac{3}{2}s)v_i$ after it, while $\Delta T = \frac{2}{3}|\Delta x|$. This gives the first row.

For u_i, both endpoints have time coordinate $2s$, so its length is $3s/2$.

For v_i^{edge}, the spatial displacement is (u_s, u_s) and the time displacement is $3u_s/5$. Hence

$$\ell^2 = u_s^2\left(2 - \frac{9}{25}\right) = \left(\frac{3\sqrt{41}}{11}s\right)^2.$$

For w_i^{-}, the spacetime displacement from B_i to C_i is

$$(\Delta T, \Delta\xi, \Delta\eta) = \frac{s}{22}(18, -3, 30).$$

Therefore

$$\ell^{2} = \frac{s^{2}}{22^{2}}(9 + 900 - 324) = \left(\frac{3\sqrt{65}}{22}s\right)^{2}.$$

The other perimeter edge is symmetric. The remaining rows have $\Delta T = 0$ and follow immediately.

Proposition (Positive face areas). *The induced areas are*

$c\|c\|c$face	$s < 0$	$s > 0$
hline V_{i}	$\frac{3}{2}$	$\frac{3}{2} - \frac{15}{22}s^{2}$
U_{i}^{-}	inactive	$\frac{9}{11}s^{2}$
U_{i}^{+}	inactive	$\frac{9}{11}s^{2}$
Q_{i}	$\frac{7}{2}$	$\frac{7}{2}$

In particular, for $0 < s \le 1$,

$$\operatorname{Area}(V_{i}) \ge \frac{9}{11} > 0.$$

Proof. On a graph plane with gradient a, induced area is Euclidean projected area multiplied by $\sqrt{1 - |a|^{2}}$.

For V_{i}, this factor is 1/3. The original projected triangle has area 9/2. Each new projected inner triangle has area

$$\frac{1}{2}a_{s}u_{s} = \frac{45}{44}s^{2}.$$

Subtracting the two inner projected areas and multiplying by 1/3 gives the stated area of V_{i}.

For either inner face, the area factor is $\sqrt{1 - 9/25} = 4/5$, giving $9s^{2}/11$. The collar quadrilateral has projected area $(16 - 9)/2 = 7/2$ and is horizontal.

One-sided edge laws

The changing boundary in the half-sector $\xi \ge \eta$ obeys

$$10\xi + \eta = 15s.$$

Its Euclidean normal velocity with respect to the parameter s is

$$\frac{15}{\sqrt{101}}.$$

In the other half-sector, the analogous equation is $\xi + 10\eta = 15s$. The rays, bisectors, and outer crest have fixed spatial directions.

The endpoint laws are

$$B_i' = \frac{3}{2}v_i, \qquad C_i' = \frac{15}{11}(v_i + v_{i+1}),$$

and the outgoing length derivatives follow directly from Proposition . Before the event, all finite edge lengths are constant.

These are slicing-parameter laws, not matter-velocity laws.

The active particle link

For $s < 0$, the link at P_s has the cyclic order

$$e_0,\ V_0,\ e_1,\ V_1,\ e_2,\ V_2.$$

For $s > 0$, its cyclic order is

$$u_0,\ U_0^-,\ v_0^{\mathrm{edge}},\ U_0^+,\ u_1,\ U_1^-,\ v_1^{\mathrm{edge}},\ U_1^+,\ u_2,\ U_2^-,\ v_2^{\mathrm{edge}},\ U_2^+.$$

The old edges e_i and faces V_i are no longer incident to the particle after the event.

The new bisector edges are genuine creases: their adjacent gradients are $(3/5, 0)$ and $(0, 3/5)$. The ray edges are also genuine creases; after unfolding across a ray, the transverse gradient changes sign. Thus the sixfold star is not produced by merely subdividing a single flat face.

Figure records one view of the same separation. Figure records one view of the same separation. Figure records one view of the same separation. The drawings add no hypothesis.

Theorem (Unique isolated particle-incidence event). *The family has exactly one incidence-event parameter in* $[-1, 1]$, *namely* $s = 0$. *At that parameter:*

1. *the six vertices* B_i, C_i *coalesce at* P_0*;*

2. *the twelve edges* $u_i, v_i^{\mathrm{edge}}, w_i^{\pm}$ *vanish;*

3. *the six faces* $U_i^{\pm}$ *vanish;*

4. *the particle link changes from three to six incident faces. There is no other collision, zero-length active edge, zero-area active face, or contact with the outer crest in the window.*

Proof. The cellulation is independent of the sign-preserving choice of s. For $s < 0$, all its finite lengths and areas are constant and positive.

For $s > 0$, the vertex formulas show that the six new vertices are distinct and lie strictly inside $\rho = 3$. Propositions and give positivity of every active finite edge and face. Lemma excludes self-intersection and omitted regions. The strict bound $\rho(C_i) \le 30/11 < 3$ excludes contact with the crest.

At $s = 0$, the stated vertices, edges, and faces collapse according to their explicit formulas, while e_i and V_i have the incoming edge and face as their nondegenerate limits. The particle-link orders displayed above differ in valence, so the event cannot be a relabelling of an unchanged star.

For reference, the finite patch counts are

$c\|ccc\|c$	vertices	edges	faces	$V - E + F$
hline $s < 0$	7	12	6	1
$s > 0$	13	24	12	1.

At $s = 0$, the regular cellulation is the incoming one. The additional outgoing labels are germ metadata, not zero-length regular edges or zero-area regular faces.

Adapted affine charts and exact holonomy

A finite rational chart table

Let

$$Q_i = \mathrm{diag}\,(1, \mathrm{Rot}\,(i\pi/2)), \qquad i = 0, 1, 2.$$

In sector coordinates define

$$B_h = \begin{pmatrix} 3 & -2 & -2 \\ -2 & 2 & 1 \\ -2 & 1 & 2 \end{pmatrix},$$

and

$$B_x = \begin{pmatrix} 5/4 & -3/4 & 0 \\ -3/4 & 5/4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad B_y = \begin{pmatrix} 5/4 & 0 & -3/4 \\ 0 & 1 & 0 \\ -3/4 & 0 & 5/4 \end{pmatrix}.$$

Direct multiplication gives

$$B_h^{\mathrm{T}} \eta B_h = B_x^{\mathrm{T}} \eta B_x = B_y^{\mathrm{T}} \eta B_y = \eta.$$

They have determinant one and positive time orientation.

For a face in sector i, use the following affine coordinate changes from the cut-space reference coordinates:

$c\|c\|c$face	$A_f(z)$	equation in the adapted chart
hline V_i	$Q_i B_h Q_i^{-1} z - 3s\, e_0$	$T' = 0$
U_i^-	$Q_i B_y Q_i^{-1} z - \frac{5}{2} s\, e_0$	$T' = 0$
U_i^+	$Q_i B_x Q_i^{-1} z - \frac{5}{2} s\, e_0$	$T' = 0$
Q_i, Z_i	z	$T' = s + 2.$

Here $e_0 = (1, 0, 0)^{\mathrm{T}}$. The same symbol Q_i for a sector rotation and a collar face is distinguished by context: $Q_i B_h Q_i^{-1}$ is a matrix expression, whereas $Q_i = [A_i, D_i, D_{i+1}, A_{i+1}]$ denotes a cell.

These charts are used on the regular part of the spacetime. Their formulas extend to face closures, but no claim is made that the cone apex has a regular affine neighbourhood.

Remark (Exact arithmetic). All linear entries are rational. All translations are rational multiples of s. Consequently, at dyadic s, every chart entry is rational and exactly representable through the displayed symbolic table. No closure assertion for an unrestricted collection of Lorentz matrices is required.

Transition words

For an oriented crossing from a face chart f to a face chart g, define the seam exponent

$$n_{gf} = \begin{cases} 1, & \text{upper seam to lower seam,} \\ -1, & \text{lower seam to upper seam,} \\ 0, & \text{a crossing not using the marked seam.} \end{cases}$$

If a chart overlap has more than one connected component, the component is part of the crossing data.

The affine transition is

$$g_{g \leftarrow f} = A_g J^{n_{gf}} A_f^{-1}.$$

This specifies every adjacency transition: ordinary adjacencies have $n = 0$, and the marked-seam adjacencies have $n = \pm 1$.

Lemma (Ordered-product identity). *For a closed chart word $f_0, f_1, \dots, f_m = f_0$,*

$$g_{f_0 \leftarrow f_{m-1}} \cdots g_{f_2 \leftarrow f_1} g_{f_1 \leftarrow f_0} = A_{f_0} J^N A_{f_0}^{-1}, \qquad N = \sum_{\ell=0}^{m-1} n_{f_{\ell+1} f_\ell}.$$

For a loop on a slice avoiding the particle, N is its oriented winding number.

Proof. Substitution of cancels every adjacent pair $A_f^{-1} A_f$ and gives the displayed product.

To identify N, lift the spatial projection of the loop to the angular cover with $\theta \in \mathbb{R}$. Each positive marked-seam crossing increases the sheet index by one, and each negative crossing decreases it by one. The terminal angular coordinate differs from the initial one by NL. Since the graph projection is a homeomorphism, this is also the winding number of the loop in the punctured slice.

Corollary (Regular products). *Every sufficiently small loop around a regular vertex has affine product equal to the identity, including regular vertices lying on the marked seam.*

Proof. Such a loop is contractible in the punctured cone and has winding number zero. Apply Lemma .

A common framed primitive meridian

Choose the collar basepoint

$$b_s = \left(s + 2, \frac{15}{8}, \frac{15}{8} \right)$$

in sector zero. Use the reference affine chart there, without re-centring its origin.

A collar meridian is the polygonal loop $\rho = 15/4$, based at b_s, traversed in increasing angular order. In the collar

face charts its ordered word is

$$I,\quad I,\quad J,$$

and therefore its affine product is exactly J.

For a loop strictly inside the moving core, choose

$$\rho = \frac{29}{10}.$$

Because

$$\frac{30}{11} < \frac{29}{10} < 3,$$

this loop lies in the V_i faces for every $s \in [-1, 1]$. Join it to b_s along the bisector of sector zero. The resulting based meridian is denoted γ_s.

Proposition (Framed affine peripheral certificate). *For every $s \in [-1, 1]$,*

$$\mathrm{Hol}\,(\gamma_s) = J.$$

The collar meridian and the core meridian are primitive and positively oriented. The same identity holds for both event germs at $s = 0$.

Proof. The angular lift of either meridian increases by exactly L, so its winding number is one.

For the core meridian, write $A_i = A_{V_i}$. Starting in the V_0 chart, the three-face product is

$$(A_0 J A_2^{-1})(A_2 A_1^{-1})(A_1 A_0^{-1}) = A_0 J A_0^{-1}.$$

The path from the collar reference chart into V_0 contributes A_0, and the return path contributes A_0^{-1}. The based product is therefore

$$A_0^{-1}(A_0 J A_0^{-1})A_0 = J.$$

This is an equality in the full affine group; the translations cancel as well as the Lorentz factors.

At $s = 0$, the core meridian avoids every collapsed cell. Its charts and transitions are common to the two germs, giving the same identity.

Holonomy through the changed particle star

The preceding common meridian provides a certificate for the whole window. The new star itself has the same nontrivial peripheral word.

For $s > 0$, the loop $\rho = s$ lies strictly inside the six-face star, because

$$h - k \le h = \frac{2}{3}s < s.$$

In angular order its charts are

$$U_0^-, U_0^+, U_1^-, U_1^+, U_2^-, U_2^+.$$

Let $A_0, \dots, A_5$ denote these six adapted coordinate changes. The ordered product is

$$(A_0 J A_5^{-1})(A_5 A_4^{-1}) \cdots (A_1 A_0^{-1}) = A_0 J A_0^{-1}.$$

Transporting the framing to the common collar again gives J.

For $s < 0$, a sufficiently small meridian meets the three V_i faces and has the three-factor product already computed. The local outgoing loop may shrink as $s \downarrow 0$; it is not used as the nonvanishing loop at the event parameter. Its correspondence with γ_s is the explicit annular homotopy in the punctured slice.

Affine axis and gauge

For an affine change of reference coordinates $A = (C, a)$,

$$AJA^{-1} = \big(CR(\mu)C^{-1}, ?a - CR(\mu)C^{-1}a\big).$$

Moreover,

$$AJA^{-1}(Az) = Az \qquad (z \in \mathscr{W}).$$

Thus the affine holonomy fixes the developed particle worldline pointwise in every presentation.

The literal equality $\mathrm{Hol}(\gamma_s) = J$ uses the declared reference framing. Under an unrestricted common affine change of framing, the invariant statement is the corresponding conjugacy class together with its marked fixed axis.

Because $J^4 = I$, a matrix product alone does not establish primitivity. A fourfold meridian is noncontractible but has identity holonomy. Primitivity in Proposition was proved by the angular lift, independently of the matrix calculation.

The particle packet and the geometric rule

Rest data and face normals

The geometric particle packet is

$$\Pi_\star = (P_\star, \mu, \mathscr{W}, u_\star = e_0, \text{cell label} P, \text{reference framing}).$$

It is the same packet on both germs.

The future unit normal of a V_i face, in sector coordinates, is

$$n_h = (3, 2, 2).$$

The future unit normals of U_i^+ and U_i^- are respectively

$$n_x = \left(\frac{5}{4}, \frac{3}{4}, 0\right), \qquad n_y = \left(\frac{5}{4}, 0, \frac{3}{4}\right).$$

They are rotated into the developed sector by Q_i.

Consequently the particle's face-relative rapidities satisfy

$$\cosh \chi_h = 3, \qquad \cosh \chi_k = \frac{5}{4},$$

so

$$\chi_h = \operatorname{arcosh} 3, \qquad \chi_k = \log 2.$$

The reference rest vector remains e_0. A stationary worldline therefore does not imply zero rapidity relative to every incident face.

Germs and two different transports

Let g_0^- be the limiting slice at $s = 0$ equipped with the incoming one-sided formulas and labels, and let g_0^+ be the same limiting slice equipped with the outgoing one-sided formulas and labels. The additional outgoing cells are inactive germ slots at $s = 0$.

The geometric particle realization transport is

$$P_0^- \mathrm{longmapsto}\, P_0^+,$$

with both points equal to $(0, 0, 0)$ on the fixed worldline.

The matter-name transport is a function on cell labels. Retain the labels

$$P,\ A_i,\ D_i,\ e_i,\ c_i,\ t_i,\ o_i,\ V_i,\ Q_i$$

and the six exterior labels. Define

$$\tau(\ell) = \ell$$

for every incoming label ℓ. The geometric realizations of some persistent labels change: for example, e_i is $[P, A_i]$ before the event and $[B_i, A_i]$ afterward. Cell-label transport is abstract location transport, not a trajectory for a material point.

Thus particle realization transport and matter-name transport are different typed maps, even though both preserve the particle-cell label P.

Exact guard

Let $s = j2^{-d}$ be the decoded clock, with

$$-2^d \le j \le 2^d.$$

Let $m = 0$ denote the incoming mode and $m = 1$ the outgoing mode. Let v be a permission counter.

The forward event domain is

$$m = 0, \qquad j = 0, \qquad v \geq 1,$$

with the declared canonical geometric data.

The rule is

$$G : (g_0^-, \Pi_\star, v, \ell_1, \ldots, \ell_n, \rho) \text{longmapsto } (g_0^+, \Pi_\star, v - 1, \tau(\ell_1), \ldots, \tau(\ell_n), \rho).$$

No coupling or measurement occurs.

Proposition (Spectator-blind control). *The guard, geometric output, particle packet, certificates, and transport τ of satisfy Contract . They are independent of the number and internal state of matter names.*

Proof. The guard reads only m, j, v and fixed geometric validity data. The output geometry is the explicitly prescribed outgoing germ. The particle packet is a fixed geometric packet. The holonomy and geometric certificates depend only on the displayed family and chart table. The transport is the fixed identity map on persistent labels.

None of these definitions reads a location occupancy count or an internal matter state. The same geometric event is defined when there are no matter names at all. Applying the same fixed map to each supplied name changes no geometric decision.

Remark (A guard is not a report comparison). The condition $j = 0$ is an exact clock test. It is not inferred from a rounded edge length. At the event, newly born edge slots have limiting length zero, but for sufficiently small positive dyadic s their rounded reports can also be zero. The particle deficit is likewise a fixed symbolic identifier, not an estimate extracted from a length report.

Exact finite decoder and reversible implementation

A finite cell dictionary

There are 25 cells in the incoming compact patch and 49 in the outgoing compact patch. The fixed exterior adds three unbounded rays and three faces. Thus the global dictionary contains 55 labels, and six bits suffice for a cell location.

One explicit dictionary is:

$c\|l$integer codes	labels
hline 0	P
1--3	A_i
4--6	D_i
7--9	e_i
10--12	c_i
13--15	t_i
16--18	o_i
19--21	V_i
22--24	Q_i
25--27	B_i
28--30	C_i
31--33	u_i
34--36	v_i^{edge}
37--39	w_i^-
40--42	w_i^+
43--45	U_i^-
46--48	U_i^+
49--51	exterior rays
52--54	Z_i.

Codes $55, \dots, 63$ are invalid.

At $s = 0$, codes $25, \dots, 48$ are germ metadata and are not valid regular locations. All incoming location labels remain valid outgoing locations at the event. In particular, τ is injective.

Three distinct numerical roles

Fix integers

$$b \geq 0, \qquad d \geq 0, \qquad r \geq 1.$$

They specify report precision, clock precision, and permission width.

The continuous family is used to prove geometry and one-sided laws. The finite decoder represents only its dyadic subfamily. Rounded reports are additional finite data and do not define that subfamily.

For each of the 24 finite edge slots in Proposition , store

$$q_e = \lfloor 2^b \ell_e \rfloor.$$

An inactive slot has report zero. All finite lengths are less than 6, so $b + 3$ bits per report suffice.

The canonical report residual satisfies

$$0 \le \ell_e - 2^{-b} q_e < 2^{-b}.$$

All reports can be determined exactly from the symbolic length table: the lengths are rational multiples of square roots of fixed integers, and floor comparisons reduce to exact rational or integer comparisons.

Code layout

The geometric control and report block has

$$M_{\mathrm{geom}} = 1 + r + (d + 2) + 1 + 24(b + 3)$$

bits, allocated as follows:

$l\|c$field	bits
hline mode$m$	1
permission$v$	r
clock$j$	$d + 2$
fixed family/chart-table identifier	1
24 length reports	$24(b + 3)$.

The clock uses signed two's-complement representation, restricted to $-2^d \le j \le 2^d$. Thus $j = 0$ is an exact integer-zero test.

The particle block has nine bits: six for its cell label and three fixed identifiers for the particle, deficit, and reference rest/worldline data. Their semantic values are the fixed packet $\Pi_\star$; no numerical approximation to $\pi/2$ is

stored.

Set

$$B = M_{\mathrm{geom}} + 9.$$

For each original matter name reserve six location bits and six history bits. These are not charged to the geometric budget.

Decoder

A canonical geometric word determines:

1. $s = j2^{-d}$;

2. the exact function ;

3. the appropriate cellulation from Section ;

4. the affine chart table from Section ;

5. the fixed particle packet;

6. the auxiliary canonical reports. For $j < 0$, only incoming mode is valid; for $j > 0$, only outgoing mode is valid. At $j = 0$, the two mode values select the two declared germs.

The decoder reconstructs coordinates and exact lengths from the symbolic formulas, not from report words. It ignores retained implementation history. Intermediate compiler states are not assigned geometric interpretations.

Proposition (Exact forward decoding). *Let f be the map on canonical B-bit geometric words at the forward event that changes*

$$m : 0 \mapsto 1, \qquad v \mapsto v - 1,$$

and leaves the clock and particle packet fixed. Its outgoing reports are the canonical limiting reports at $s = 0$*. Then, on the forward event domain, decoding f gives precisely the geometric part of* .

Proof. At $s = 0$, all persistent finite edges have identical incoming and outgoing limiting lengths. Every new slot has limiting length zero. Thus the two canonical limiting report arrays agree. The mode change selects the outgoing germ rather than the incoming germ, and the permission field has the required decrement. All other decoded data agree with .

A finite reversible extension

Although the forward map in this example is already injective, we retain complete input history. This gives a simple, explicit implementation certificate and makes no minimal-memory claim.

Let D_{geo} be the finite set of canonical forward geometric words. For the fixed family, its only free geometric field is $v \in \{1, \dots, 2^{r} - 1\}$, so

$$|D_{\mathrm{geo}}| = 2^{r} - 1.$$

On two B-bit blocks prescribe

$$(x, 0_{B}) \mathrm{longmapsto} \, (f(x), x), \qquad x \in D_{\mathrm{geo}}.$$

Inputs and outputs are disjoint because their first-block mode bits are respectively zero and one. Distinct inputs have distinct outputs because the second output block retains x.

Define a permutation P_{geo} on all $2B$-bit words by exchanging each pair

$$(x, 0_{B}) \ \leftrightarrow \ (f(x), x)$$

and fixing every other word.

Lemma (Exact reversible synthesis). *A permutation of N logical bits consisting of q specified transpositions can be implemented using NOT, CNOT, and Toffoli gates, with* $N - 2$ *clean work bits returned to zero. For the transpositions above, a sufficient gate bound is*

$$4N(2N - 1)q.$$

Proof. Connect the two words of a transposition by a shortest Hamming path of length $k \le N$. The endpoint transposition is a product of $2k - 1$ adjacent-word transpositions along that path.

An adjacent-word transposition is a NOT on the differing bit controlled on the specified pattern of the other $N - 1$ bits. Negative controls are obtained by conjugating the relevant controls with NOT gates. The positive multiple-control NOT is implemented by computing the control conjunction into clean work bits with Toffoli gates, performing the target flip, and reversing the conjunction computation. At most $N - 2$ work bits suffice. For $N \ge 3$, fewer than $4N$ elementary gates suffice for each adjacent-word transposition, including the NOT conjugations.

Applying this construction to all q transpositions gives the bound. Every conjunction computation is uncomputed, so the work bits return to zero.

Only this elementary finite-permutation argument is needed; no efficiency claim is made.

Separate name storage and spectator-independent geometry

Apply Lemma to P_{geo} with

$$N_{\mathrm{geo}} = 2B, \qquad q = 2^{r} - 1.$$

This geometric circuit is independent of the number of names.

For a supplied list of n original names, append six location and six history bits per name. On the forward domain the location transport is the identity. After the geometric circuit, copy each location bit to its initially zero history bit, controlled by the outgoing mode bit. This uses six Toffoli gates per name and never feeds a name bit into a geometric output.

A sufficient total bit count, including clean work, is

$$\text{boxed}\, M = 2B + 12n + (2B - 2) = 4B - 2 + 12n.$$

A sufficient gate count is

$$4N_{\mathrm{geo}}(2N_{\mathrm{geo}} - 1)(2^{r} - 1) + 6n.$$

The geometric budget is independent of n. The statement concerns a family of finite circuits, one for each fixed finite name list, not a single bounded circuit accepting unboundedly many names.

Theorem (Forward-domain implementation). *For every $b, d \geq 0$, $r \geq 1$, and fixed finite original-name list, there is an exact NOT–CNOT–Toffoli circuit U with the bit bound such that*

$$\mathrm{Dec}\,(Uc) = G(\mathrm{Dec}\,(c))$$

for every canonical forward event code c with clean history and work registers.

The circuit retains input history, returns all work bits to zero, and acts on the internal matter space as

$$U_{\mathrm{code}} \otimes \mathrm{id}_{\mathscr{H}_n}.$$

Its geometric output and geometric control are independent of spectator number and spectator state.

Proof. The geometric permutation sends $(x, 0_B)$ to $(f(x), x)$ on the forward domain. Proposition identifies its decoded geometric output with the geometric part of G.

The location registers are unchanged, which implements τ. Each name history receives its original location. The copied histories are ignored by the decoder. Lemma returns geometric work to zero, and the name-copy gates require no additional work.

The geometric circuit acts only on geometric and geometric-history registers. The subsequent name-copy gates do not modify any geometric register. No gate acts on the internal matter factor.

The asserted decoder identity is restricted to the declared forward domain. No geometric meaning is assigned to intermediate gate states or to arbitrary inputs of the permutation completion.

Remark (History and observations). History is implementation storage, not a measurement record. Its presence does not enlarge the observation algebra of the geometric rule. Since the present forward map is injective, some of this storage can be removed by a more economical construction; such optimization is irrelevant to existence of the

certificate.

The certificate theorem

Theorem (Particle-supported slicing certificate). *For the fixed spinless cone with*

$$\mu = \frac{\pi}{2}, \qquad \mathscr{W} = \{(T, 0, 0)\},$$

the family

$$F_s = s + \min\left\{2, \max\left[\frac{2}{3}(\xi + \eta), s + \frac{3}{5}\min\{\xi, \eta\}\right]\right\}, \qquad -1 \le s \le 1,$$

defined sectorwise on the three identified quadrants, has the following certificate.

1. *The formulas define one continuous single-valued graph on the cone, not three incompatible developments.*

2. *The slices are embedded and uniformly spacelike with $\lambda = 2\sqrt{2}/3$, and strictly ordered with $c = 1$.*

3. *There is exactly one incidence event, at $s = 0$. The particle is in the changing incidence structure, and its link changes from three to six incident faces.*

4. *Vertices, faces, one-sided edge laws, finite edge lengths, positive face areas, and collapsed germ slots are given exactly in Section .*

5. *All regular local affine products are the identity. A declared primitive positively oriented meridian has framed affine product J on both germs and throughout the parameter window. In every changed affine framing, the conjugate holonomy fixes the corresponding developed particle worldline pointwise.*

6. *The compact patch matches one prescribed exterior through an annular collar disjoint from the worldline. The collar and the moving core have the same peripheral affine holonomy.*

7. *The particle packet is preserved. Particle realization transport and matter-name cell transport are separately defined.*

8. *The event guard and geometric output satisfy spectator-blind control. No occupancy test, coupling, or*

measurement is used.

9. *The finite decoder reconstructs exact dyadic family members from symbolic data. Reports are auxiliary, and the event guard tests the stored integer* $j = 0$.

10. *For every fixed finite original-name list, the forward event has an exact reversible implementation with retained history, uncomputed work, and identity action on internal matter. Consequently this is a particle-supported G-rule certificate for Contract and Definition .*

Proof. Item (1) follows from the matching values of ρ, h, k on all identified rays. Item (2) is Proposition . Items (3) and (4) are Theorem and Propositions –.

Item (5) follows from Lemma , Corollary , Proposition , the explicit six-face star product, and . Item (6) follows from $F_s = 2 + s$ for $\rho \geq 3$ and the common peripheral calculation.

Item (7) is the construction in Section . Item (8) is Proposition . Item (9) is the decoder construction and Proposition . Item (10) is Theorem . No new geometric assertion is required in this assembly step.

Comparison with a vacuum exchange

A contractible vacuum patch has trivial peripheral winding and therefore trivial peripheral affine product in a compatible flat development. Here the updated particle star has primitive winding one and affine product $J \neq I$.

The distinction is not merely that a particle lies somewhere in a large declared region. The particle is the common endpoint of the emerging edges, and its incidence valence changes. The outgoing six-face product explicitly includes the same nontrivial identification that appears in the collar.

The construction therefore places the conical identification inside the changing geometric interface while preserving the stated observation boundary.

Negative examples and precise limitations

Seam mismatch

Lemma proves a restricted obstruction for the axis-aligned vacuum maximum with unequal slopes. Changing a report table or adding a cap cannot repair a failure of single-valuedness near the apex.

Padded support

A vacuum exchange performed in a small contractible region remains a vacuum exchange if one merely declares a larger support containing a remote particle. It does not meet the active-incidence condition: the particle link is unchanged and is not incident to the cells that collapse or emerge.

Theorem excludes this failure directly.

Fake particle assignment

A label called ``particle'' at a regular vacuum vertex does not produce a conical particle. If all small peripheral products at that vertex are the identity and the geometric realization is regular, the label has not supplied the missing conical identification.

Here the affine product is computed from seam-compatible charts, and the marked singular axis is fixed by that product.

Correct Lorentz part, incorrect affine part

For $\delta \neq 0$, the affine map

$$J_{\delta}(z) = R(\mu)z + \delta e_{0}$$

has the same Lorentz part as J, but it does not fix the particle worldline pointwise. Indeed,

$$J_{\delta}(T, 0, 0) = (T + \delta, 0, 0).$$

Its time translation cannot be removed by conjugating it to a spinless rotation with a fixed timelike axis, since it has no fixed point.

This algebraic example explains why the certificate checks the whole affine word rather than only its Lorentz part. Time-shifted identifications also appear in the classical discussion of spinning sources [1].

What an interface obstruction would require

Proposition (Observation indistinguishability obstruction). *Suppose a rule's geometric output must factor through a declared observation map* Obs*. If there exist admissible configurations* x, x' *such that*

$$\mathrm{Obs}\,(x) = \mathrm{Obs}\,(x')$$

but the target requires different geometric outputs at x *and* x'*, then no rule with that observation boundary realizes the target on both configurations.*

Proof. If the output is $\Phi(\mathrm{Obs}\,(x))$, equality of the observations forces equality of the outputs. This contradicts the target requirement.

Failure of one proposed occupancy-based guard does not establish this proposition's hypothesis for every implementation. Nor does noninjective location transport alone imply an obstruction when sufficient history is allowed.

Limits of the result

The spacetime, deficit, and particle worldline are fixed. The event changes a prescribed slicing and its cell incidence. It does not derive the particle trajectory from matter data.

The affine chart table is finite and explicit. The circuit implements a finite coded event, not the entire continuous family and not an unrestricted Lorentz computation.

No computational metric or scheduling theorem is supplied. The physical diameter of the compact patch is not a substitute for such a metric.

Finally, this example has four-valent regular vertices and nonconvex faces. It is not claimed to lie in every stricter polygon calculus; see Remark . Its exact content is the certificate of Theorem .

appendix

Intrinsic-angle audit

This appendix distinguishes the spacetime deficit μ from intrinsic slice defects.

For a vertex v in a polyhedral slice, write

$$\kappa_v = 2\pi - \sum_{f \ni v} \theta_{f,v},$$

where $\theta_{f,v}$ is the intrinsic face angle.

Proposition (Particle slice defects). *The particle's intrinsic slice defects are*

$$\kappa_P^- = 2\pi - 3\arccos\left(-\frac{4}{5}\right) \qquad (s < 0),$$

and

$$\kappa_P^+ = 2\pi - 6\arctan\left(\frac{4}{5}\right) \qquad (s > 0).$$

Neither formula identifies the particle's slice defect with the fixed spacetime deficit $\mu = \pi/2$.

Proof. On an incoming V_i face, the induced metric in sector coordinates is

$$G_h = I - \begin{pmatrix} 2/3 \\ 2/3 \end{pmatrix}\begin{pmatrix} 2/3 & 2/3 \end{pmatrix}.$$

For the two ray directions,

$$\cos\theta_h = \frac{-4/9}{(\sqrt{5}/3)^2} = -\frac{4}{5}.$$

Three such angles meet at the particle.

On U_i^-, the induced metric is

$$G_k = \operatorname{diag}(1, 16/25).$$

The angle between $(1,0)$ and $(1,1)$ is $\arctan(4/5)$. The same angle occurs on U_i^+. There are six such angles.

Proposition (Total-angle audit). *For every slice in the family, summing over all interior vertices of the compact patch gives*

$$\sum_{v \in \operatorname{int} K_s} \kappa_v = \frac{\pi}{2}.$$

At $s = 0$*, only regular cells of the limiting slice enter the sum.*

Proof. Triangulate the polygonal faces without changing their metric. The Euclidean angle sum in each triangle, together with Euler's formula for a disk, gives

$$\sum_{v \in \operatorname{int} K_s} \kappa_v + \sum_{v \in \partial K_s} (\pi - \theta_v) = 2\pi.$$

Artificial triangulation edges do not change the summed angles.

The boundary lies in the resting collar. At each of the three boundary vertices D_i, the total interior angle is $\pi/2$, so the boundary turning sum is $3\pi/2$. Therefore the interior defect sum is $\pi/2$.

The individual particle defect changes across the event, while defects at the other interior vertices compensate. At the event, several of those vertices coalesce. This intrinsic accounting does not replace the affine holonomy calculation.

Implementation bookkeeping

For clarity, the bit budget can be written without abbreviations:

$$M_{\text{geom}} = 24b + d + r + 76, \qquad B = 24b + d + r + 85,$$

and hence

$$M = 96b + 4d + 4r + 338 + 12n.$$

This includes:

• the current geometric and particle data;

• a full geometric and particle history block;

• six location and six history bits per original matter name;

• $2B - 2$ clean work bits.

Only the 24 finite edge slots of the compact patch are reported. The three unbounded exterior rays have no finite length report. Their exact geometry is fixed by the exterior cellulation.

The use of full histories is deliberately redundant. It separates three statements that should not be conflated:

1. the geometric event exists;

2. its declared forward code map is decoder-correct;

3. that map has a reversible finite extension. No interpretation of off-domain compiler states is needed for any of these statements.

Book expansion notes (t433u)

The three-to-six particle-supported certificate is local to the prescribed cone. VIII-A affine matching is a comparison, not a premise. A stale deficit rotation is not this slicing. Observation-boundary source record: [10].

Standing ledger. Not spacetime emergence; not Einstein dynamics; not a Page curve, island, bounce, vacuum-energy selection, or de Sitter state as an Enter theorem; not a derived metric. O.1 stays NOT_CLOSED. XVI-N is not discharged. XV-B is not opened. No quantum coefficient is admitted. Host C is not created. $\mathrm{Dom}(T)$ remains undetermined; J2 remains unresolved. Problems 6, 7, and 9 are named, not solved. Gate 0 and ST4b stay NOT_SUPPLIED. Host-A stays NAMED_COMPARISON_ONLY $\{\mathrm{GI}, \mathrm{CB}\}$. The shredder is not Gibbons–Hawking. Cited sheets are source records, not premises. Conjectures stay conjectures. wired=false.

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