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Landmark VII-G Completely positive observation boundaries for finite matter–geometry feedback

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Abstract

We replace an unrestricted deterministic state-functional coupling by a finite classical–quantum transition instrument. Classical labels contain the geometric interface and every controller variable used to select future operations. Matter occupies a fixed finite-dimensional Hilbert space, and the unconditional evolution is linear, completely positive, and trace preserving.

We give an exact observation criterion in the direct-sum state algebra and strengthen it to agreement of stochastic history laws and history-conditioned accessible residuals. A finite backward construction identifies the smallest observable linear spaces needed for a declared continuation. Arbitrary entangled spectator extensions preserve every unnormalized geometric-history block after restriction, provided the complete controller and its certificates are spectator blind. Implementation equivalence is characterized by selective adjoint maps and is stable under compatible continuations.

Two limitations are proved independently: a nonconstant deterministic packet selection cannot be the classical output of an affine channel on a convex state domain, and an informative classical record is incompatible with exact recovery of every input state. Resource and energy balance are stated as operator identities, with resource accounting kept distinct from physical energy. Choi bounds yield checkable assisted stability estimates for both terminal states and complete histories.

A resource-budgeted unsharp qubit instrument selects strictly ordered horizontal slices of a fixed spinless cone. Every branch has an explicit certificate; its classical sector description and a quantitative dephasing approximation are calculated exactly. This example is matter-controlled slicing, not matter-driven gravitational dynamics. No quantum-field source, changing deficit, moving particle, or gravitational field equation is constructed.

Bibliographic record

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The finite problem and its relation to the series

The nonlinear benchmark

The observation interface of VII-E allows a coupling rule of the form

$$g' = F_\nu(g, \omega_\rho|_{\mathcal{A}_\nu}),$$

while leaving the matter state unchanged. VII-E explicitly notes that this need not be affine and is not thereby a quantum channel [6, Definition def:rules and Remark rem:nonlinear].

A concrete benchmark is the rule called *Option N* in VIII-B:

$$\mu_{\theta}(\rho) = \mu_{0} + \theta\kappa I_{f}(\rho), \qquad \rho \operatorname{longmapsto} \Pi_{A}(\mu_{\theta}(\rho)).$$

Its domain requires the selected deficit to stay in the declared interval. The rule selects one definite geometric packet and does not acquire a stochastic measurement record. The finite source letter of VIII-C uses a supplied source token in a prescribed table-selection command, including the word

$$W = \mathrm{G};\ \mathrm{Stab};\ \mathrm{G};\ ?\mathrm{E}_{\mathrm{particle}};\ \mathrm{G}.$$

A promised token is not an instrument that extracts an expectation from unknown matter [8, 9].

We take these as named comparison targets, not as premises providing a joint completely positive evolution. Proposition below identifies exactly why a nonconstant deterministic selection cannot simply be promoted to an unconditional affine channel.

Established framework and present contribution

Quantum instruments distinguish outcome probabilities from their selective residual maps [11]. Finite-dimensional complete positivity and its matrix certificates are standard [12, 13]. Linear completely positive classical–quantum dynamics is also an established framework, including stochastic trajectories and broader hybrid generators [16, 17, 18].

Accordingly, we do not claim novelty for the direct-sum state space, the Kraus construction, the exact recovery–complement criterion, or the general channel observation principle. In particular, VII-E already proves a composition criterion for block-diagonal state spaces [6, Proposition prop:classical-composition]. Our direct-sum criterion is its intrinsic specialization.

The purpose of VII-G is to organize these facts into a complete finite feedback specification, with:

1. a state-independent transition table and certified geometric output graph;

2. branch-sensitive observation boundaries and stochastic histories;

3. exact consistency with entangled spectators;

4. selective-map implementation tests;

5. resource accounting and assisted error certificates;

6. an explicit finite example satisfying the specification. All results used in the assembly are proved below.

Separation from gravitational dynamics

A geometric label can describe a slice, a germ, a controller state, or a spacetime parameter. Updating such a label is not, by itself, a gravitational evolution law. The distinction is

$$\text{boxed matter-controlled slicing} \neq \text{matter-driven gravitational dynamics}.$$

VII-F supplies a demanding fixed-cone incidence certificate, including full affine holonomy and exact event control, but keeps its spacetime, deficit, and worldline fixed [7]. We use its certificate discipline, not an unproved extension of its geometry. The example in Section is simpler: its slices are horizontal and its particle link does not change.

The field-theoretic manuscripts VII-A–D are not prerequisites of the finite results. Their relevance to a later source construction is discussed in Section .

Hybrid states, controllers, and transition instruments

Direct-sum conventions

Fix a nonempty finite label set G and a Hilbert space $\mathscr{H}$ of dimension $d < \infty$. Set

$$\mathfrak{X}_{\mathrm{G}} = \operatorname{bigoplus}_{g \in \mathrm{G}} L(\mathscr{H}).$$

For $X = (X_g)_g$ and $B = (B_g)_g$, use

$$\operatorname{Tr}_{\mathrm{G}} X = \sum_g \operatorname{Tr} X_g, \qquad \langle B, X \rangle = \sum_g \operatorname{Tr}(B_g X_g).$$

The identity is $1 = (I)_g$. The relevant norms are

$$\|X\|_{1,\mathrm{G}} = \sum_g \|X_g\|_1, \qquad \|B\|_{\infty,\mathrm{G}} = \max_g \|B_g\|_\infty.$$

A hybrid state is a positive element $\rho = (\rho_g)_g$ with $\mathrm{Tr}_G \rho = 1$. Equivalently,

$$\sigma_\rho = \sum_g \mathrm{ketbra}\, gg \otimes \rho_g.$$

The blocks are *subnormalized*:

$$p_g = \mathrm{Tr}\, \rho_g, \qquad \hat{\rho}_g = \rho_g / p_g \quad \text{only if} p_g > 0.$$

Conditional normalization is not used in the unconditional evolution law.

We use

$$d_{\mathrm{tr}}(\rho, \sigma) = \tfrac{1}{2}\|\rho - \sigma\|_{1,G}, \qquad d_{\mathrm{TV}}(p, q) = \tfrac{1}{2}\sum_x |p_x - q_x|, \qquad d_\diamond(\Phi, \Xi) = \tfrac{1}{2}\|\Phi - \Xi\|_\diamond.$$

The classical-input meaning of the diamond norm is made precise in Section .

A declared controller, not a state-dependent compiler

The label g contains every classical variable used to select subsequent operations. A typical label is

$$g = (\gamma, \mathrm{clock}, \mathrm{permission}, \mathrm{controller\ memory}, \mathrm{declared\ records}),$$

where γ denotes geometric data.

An operation table may depend on g, but not on an undeclared functional of the current unknown density operator. Adaptive measurement control is represented by recording the outcome in a classical successor label. If a finer outcome affects future choices, it must remain in the controller state even when a coarser geometric report is displayed.

For a fixed finite horizon, any finite instruction tree can be made Markovian by adjoining its node or relevant history to the label. This may require exponentially many labels. No bounded-memory claim for arbitrary horizons is implied.

Definition (Certified transition graph). A certified finite interface consists of:

1. the finite labels G, each with a declared admissible geometric realization and controller interpretation;

2. a directed edge relation $\mathscr{E} \subseteq \mathrm{G} \times \mathrm{G}$;

3. a fixed certificate for each edge $(g, h) \in \mathscr{E}$, specifying all affected output data, matching requirements, controller updates, and any location transport;

4. a declared reserve for implementation history and spectators, not inspected by geometric control. The certificates are classical data. The definition does not prove a gravitational constraint equation.

The channel construction

For each $g, h \in \mathrm{G}$, let

$$\Psi_{h|g} : L(\mathscr{H})\mathrm{longrightarrowL}\,(\mathscr{H})$$

be completely positive. Require

$$\sum_{h} \Psi^{*}_{h|g}(I) = I \qquad (g \in \mathrm{G}).$$

Nonzero branches are permitted only on certified edges.

Define

$$(\mathfrak{F}\rho)_{h} = \sum_{g} \Psi_{h|g}(\rho_{g}).$$

Here and below the adjoint is characterized by $\mathrm{Tr}\,(B\Psi(X)) = \mathrm{Tr}\,(\Psi^{*}(B)X)$.

Theorem (Finite hybrid channel structure). *The map is completely positive and trace preserving on $\mathfrak{X}_{\mathrm{G}}$. It preserves the classical–quantum state space and has adjoint*

$$(\mathfrak{F}^{*}B)_{g} = \sum_{h} \Psi^{*}_{h|g}(B_{h}).$$

Conversely, every completely positive trace-preserving map between two finite direct sums has this branch form. If

$$\Psi_{h|g}(X) = \sum_{\alpha} K_{h|g,\alpha} X K^{\dagger}_{h|g,\alpha},$$

then

$$\sum_{h,\alpha} K^{\dagger}_{h|g,\alpha} K_{h|g,\alpha} = I.$$

A canonical extension to the full matrix space on $\mathbb{C}^{\mathrm{G}} \otimes \mathscr{H}$ *has Kraus operators*

$$\text{ketbra}\, hg \otimes K_{h|g,\alpha}.$$

This extension discards off-diagonal input label blocks and has block-diagonal output.

Proof. After adjoining any matrix ancilla, a positive direct-sum input has positive blocks. Each amplified branch map is positive, and sums of its outputs are positive. Thus $\mathfrak{F}$ is completely positive. Furthermore,

$$\mathrm{Tr}_{\mathrm{G}}\, \mathfrak{F}(X) = \sum_{g} \mathrm{Tr}\left(\left[\sum_{h} \Psi^{*}_{h|g}(I) \right] X_g \right) = \mathrm{Tr}_{\mathrm{G}}\, X.$$

The block formula proves preservation of classicality, and pairing with B gives .

For the converse, compose the given map with the completely positive injection into input block g and projection onto output block h. These compositions define completely positive $\Psi_{h|g}$. Linearity gives the branch formula, and trace preservation on each input block gives .

Finite-dimensional completely positive maps have Kraus representations: the positive Choi matrix can be decomposed into rank-one positive terms and each vector unvectorized into a Kraus operator [12]. Completeness gives . The operators have sum of squared moduli equal to the full identity. Their action reads only the diagonal input blocks and writes only diagonal output blocks, proving the final assertion.

Proposition (Composition). *If a second transition has branches* $\Lambda_{k|h}$*, the composed channel has branches*

$$\Omega_{k|g} = \sum_{h} \Lambda_{k|h} \circ \Psi_{h|g}.$$

Its certificates are concatenations of the declared edge certificates.

Proof. Substitution into the two block formulas gives . Sums and compositions preserve complete positivity. Moreover,

$$\sum_{k} \Omega^{*}_{k|g}(I) = \sum_{h} \Psi^{*}_{h|g}\left(\sum_{k} \Lambda^{*}_{k|h}(I) \right) = \sum_{h} \Psi^{*}_{h|g}(I) = I.$$

The certificate assertion means concatenation of already declared certificates; it introduces no new geometric existence claim.

Observation boundaries and stochastic histories

Observable linear spaces

Definition (Observation space). An observation space is a complex linear, adjoint-closed subspace $\mathcal{O} \subseteq \mathfrak{X}_{\mathrm{G}}$ containing 1. Thus it is a finite-dimensional operator system. It need not be closed under multiplication.

Two states agree on $\mathcal{O}$ if

$$\langle B, \rho \rangle = \langle B, \sigma \rangle \qquad (B \in \mathcal{O}).$$

A frequently used choice is

$$\mathcal{O}_{\mathcal{A}} = \mathrm{bigoplus}_{g} \mathcal{A}_{g},$$

where each $\mathcal{A}_{g}$subseteqL ($\mathcal{H}$) is a unital ^*-subalgebra. This choice observes the complete classical label, because it contains each central block projection.

Use of linear spaces is important: the smallest future-sensitive observation space constructed below need not be an algebra.

Theorem (Intrinsic hybrid observation criterion). *Let $\mathfrak{F} : \mathfrak{X}_{\mathrm{in}} \to \mathfrak{X}_{\mathrm{out}}$ be a channel and let $\mathcal{O}_{\mathrm{in}}, \mathcal{O}_{\mathrm{out}}$ be observation spaces. The following are equivalent:*

1. *for all hybrid states,*

$$\rho|_{\mathcal{O}_{\mathrm{in}}} = \sigma|_{\mathcal{O}_{\mathrm{in}}} \Rightarrow ?\mathfrak{F}(\rho)|_{\mathcal{O}_{\mathrm{out}}} = \mathfrak{F}(\sigma)|_{\mathcal{O}_{\mathrm{out}}};$$

2.

$$\mathfrak{F}^{*}(\mathcal{O}_{\mathrm{out}}) \subseteq \mathcal{O}_{\mathrm{in}}.$$

For the direct-sum spaces $\mathcal{O}_{\mathrm{in}} = \mathrm{bigoplus}_{g} \mathcal{A}_{g}$ and $\mathcal{O}_{\mathrm{out}} = \mathrm{bigoplus}_{h} \mathcal{B}_{h}$, this is equivalent to

$$\Psi^{*}_{h|g}(\mathscr{B}_h) \subseteq \mathscr{A}_g \qquad (g,h).$$

Proof. The inclusion implies agreement by pulling each output observable back to the input space.

Conversely, let $H = H^{\dagger} \in \mathfrak{X}_{\text{in}}$ annihilate the Hermitian part of $\mathscr{O}_{\text{in}}$ under the trace pairing. Because $1 \in \mathscr{O}_{\text{in}}$, $\operatorname{Tr} H = 0$. Choose a faithful hybrid state ω; a multiple of the block identity suffices. For sufficiently small $a > 0$, $\omega \pm aH$ are states agreeing on $\mathscr{O}_{\text{in}}$. The assumed output agreement implies

$$\langle \mathfrak{F}^{*} B, H \rangle = 0 \qquad (B = B^{\dagger} \in \mathscr{O}_{\text{out}}).$$

Finite-dimensional double orthogonal complementation places $\mathfrak{F}^{*} B$ in the Hermitian part of $\mathscr{O}_{\text{in}}$. Complex linearity gives .

If the spaces are direct sums, an output observable may have one nonzero block B_h. Formula then makes imply every . The reverse implication follows by summing those branch images.

This is the direct-sum specialization of VII-E's composition principle and block-diagonal refinement, proved here without an ambient pinching notation [6, Theorem thm:composition and Proposition prop:classical-composition].

One-step geometric effects

Define

$$E_{h|g} = \Psi^{*}_{h|g}(I).$$

For a supported input block,

$$p(h \mid g, \hat{\rho}_g) = \operatorname{Tr}(E_{h|g} \hat{\rho}_g).$$

Corollary (Exact source restriction). *For a fixed g, all successor probabilities depend only on the state functional on $\mathscr{A}_g$ if and only if*

$$E_{h|g} \in \mathscr{A}_g \qquad (h \in \mathrm{G}).$$

Proof. Sufficiency is . For necessity, apply the faithful-state perturbation proof of Theorem to each effect.

This is the effect-pullback observation of VII-E with the successor geometry as the classical outcome [6, Theorem thm:pullback]. It says nothing by itself about the residual matter state.

History instruments

Allow a finite sequence of channels with branches $\Psi^{(t)}_{h|g}$, $t = 1, \dots, T$. All tables are fixed; the current label already contains the controller memory.

For a history $\gamma = (g_0, \dots, g_n)$, define

$$\rho_\gamma = \Psi^{(n)}_{g_n|g_{n-1}} \circ \cdots \circ \Psi^{(1)}_{g_1|g_0}(\rho_{g_0}).$$

Its probability is $p_\gamma = \operatorname{Tr} \rho_\gamma$. The complete history-retaining output is

$$\mathcal{H}_n(\rho) = \operatorname{bigoplus}_{\gamma \in \mathrm{G}^{n+1}} \rho_\gamma.$$

Forbidden paths have zero blocks.

Lemma (Completeness of history retention). *The map $\mathcal{H}_n$ is a channel. Summing its blocks with a common terminal label gives the ordinary terminal hybrid state.*

Proof. Each path map is completely positive. Summing probabilities over the final label uses ; repeating this sum backwards over the path leaves $\sum_{g_0} \operatorname{Tr} \rho_{g_0} = 1$. This proves trace preservation. The terminal summation is precisely repeated substitution in .

Theorem (History-sensitive observation principle). *Suppose that, for each t, g, h,*

$$\Psi^{(t)*}_{h|g}(\mathcal{O}_{t,h}) \subseteq \mathcal{O}_{t-1,g},$$

*where each $\mathcal{O}_{t,g}$*subseteqL *($\mathscr{H}$) contains I. If two initial hybrid states agree on* $\operatorname{bigoplus}_g \mathcal{O}_{0,g}$*, then:*

1. *all geometric history probabilities through time T agree;*

2. *for every history ending at g_n, the corresponding unnormalized blocks agree on $\mathcal{O}_{n,g_n}$;*

3. *on every positive-probability history, the normalized conditional states agree on the same space.*

Proof. For $B \in \mathcal{O}_{n,g_n}$, its history-conditioned expectation is

$$\mathrm{Tr}\,(B\rho_\gamma) = \mathrm{Tr}\left(\left[\Psi^{(1)*}_{g_1|g_0} \circ \cdots \circ \Psi^{(n)*}_{g_n|g_{n-1}}(B)\right]\rho_{g_0}\right).$$

Successive applications of put the bracket in $\mathcal{O}_{0,g_0}$. Initial agreement therefore proves (ii). Taking $B = I$ proves (i). Equal positive probabilities permit division by the same scalar, proving (iii).

The theorem compares correlated stochastic histories, not merely one-step probabilities or terminal marginals.

The smallest boundary for a fixed continuation

Proposition (Finite backward observation construction). *Fix terminal observation spaces $\mathcal{O}_{T,g}$. Define backwards*

$$\mathcal{O}_{t-1,g} = \mathrm{span}_{\mathbb{C}}\left(\{I\} \cup \bigcup_h \Psi^{(t)*}_{h|g}(\mathcal{O}_{t,h})\right).$$

These are observation spaces. They are the smallest family containing the terminal spaces and satisfying .

Agreement on the initial direct sum is equivalent to agreement of all complete length-T history-conditioned terminal expectations. It consequently implies agreement of all shorter history laws.

Proof. Adjoints of completely positive maps preserve adjoints. Hence the displayed span is adjoint closed and contains I. Any family satisfying the inclusions must contain the indicated span at each backward step, proving minimality by induction.

The pullbacks of all length-T history-conditioned terminal observables span the recursively constructed spaces. The inserted identity adds no independent expectation: at each step it equals $\sum_h \Psi^{(t)*}_{h|g}(I)$. Thus agreement of all those expectations is exactly agreement on the initial spaces. Summing extensions of a shorter history gives its probability.

For a time-homogeneous table and geometry-only terminal observations, one can also construct the boundary for every finite history.

Corollary (Finite stabilization of the all-history boundary). *Set $\mathcal{O}^{(0)}_g = \mathbb{C}I$ and*

$$\mathcal{O}_g^{(n+1)} = \operatorname{span}_{\mathbb{C}}\Big(\mathcal{O}_g^{(n)} \cup \bigcup_h \Psi_{h|g}^*(\mathcal{O}_h^{(n)})\Big).$$

The family stabilizes after at most $|G|(d^2 - 1)$ *strict enlargements. Its stable value is the smallest branch-invariant observation family containing the classical effects. Agreement on its direct sum is equivalent to agreement of all finite geometric history laws.*

Proof. The dimensions are nondecreasing and their sum is at most |Grvertd^2, starting at |G|. If one iteration does not enlarge the family, all subsequent iterations are identical. This proves the bound and branch invariance.

The recursively generated elements are spans of finite path effects. Conversely, every finite path effect appears at some iteration. The equivalence follows from the trace pairing with those effects.

No algebra closure is asserted for these minimal spaces. Replacing them by generated ^*-algebras is permissible but may declare more information accessible than the histories actually require.

The negative observation test

Proposition (Indistinguishability obstruction). *If two admissible inputs agree on the declared observation space but a target requires different accessible output expectations, no channel satisfying the corresponding adjoint inclusion realizes that target on both inputs.*

Proof. Theorem forces equality of the output expectations.

This is the linear-channel form of the observation-factorization obstruction in VII-F [7, Proposition prop:obs-obstruction]. A capacity check or rounded report is an additional observation, not a consequence of a declared matter source algebra.

Entangled spectators and complete controller blindness

Let N be a finite untouched spectator. An initial extension satisfies

$$\operatorname{Tr}_N \hat{\rho}_g = \rho_g \qquad (g \in G).$$

It may be entangled within every block.

Assumption (Blindness of the complete transition specification). Adding the spectator does not change:

1. the label dictionary or any input-label interpretation;

2. the selected branch maps, except for tensoring with id_N;

3. guard results, successor labels, certificates, or transport of original names;

4. controller updates or the declared resource reserve. Spectator storage and any spectator location history are separately reserved. Spectator outcomes are not read by the controller.

The assumption concerns the whole transition specification. Equality of one displayed source functional is not sufficient.

Lemma (Restriction of a selective map). *For every linear map Ψ on the original matter,*

$$\mathrm{Tr}_N[(\Psi \otimes \mathrm{id}_N)(X)] = \Psi(\mathrm{Tr}_N X).$$

No trace-preservation or product-state assumption is needed.

Proof. Pair both sides with an arbitrary output operator B. Their pairings equal

$$\mathrm{Tr}\,((\Psi^*(B) \otimes I_N)X).$$

Equality against all B proves the statement.

Theorem (Spectator consistency for stochastic histories). *Under Assumption and , extend every branch by $\Psi^{(t)}_{h|g} \otimes \mathrm{id}_N$. Then, for every finite history,*

$$\mathrm{Tr}_N \widehat{\rho}_\gamma = \rho_\gamma.$$

Consequently:

***1.** the complete geometric history distributions agree;*

***2.** the reduced history–matter outputs agree;*

3. *on every supported history, the conditional original-matter reductions agree;*

4. *the nonselective spectator marginal is unchanged.*

Proof. Apply Lemma to each map in . This proves without normalizing any branch. Taking traces proves (i); taking the direct sum proves (ii); division by a common positive trace proves (iii).

For (iv), the sum of all branches of a one-step transition is trace preserving on each input label. Pair the spectator reduction with B_N. Its expectation after the step is

$$\sum_g \operatorname{Tr}\left(\left[\sum_h \Psi^*_{h|g}(I)\right] \otimes B_N \,\widetilde{\rho}_g\right) = \sum_g \operatorname{Tr}\left((I \otimes B_N)\widetilde{\rho}_g\right).$$

Iteration proves nonselective invariance.

This extends the partial-trace mechanism of VII-E to selective branches and their stochastic correlations [6, Lemma lem:partial-trace, Theorem thm:spectators, and Assumption ass:blind]. It does not identify two different spectator extensions as joint outputs.

Example (A capacity leak). Suppose an update is allowed only when the total number of matter names is at most K. An original interface with K names passes. Adding one product spectator makes the same geometric update fail, even if every source algebra is $\mathbb{C}I$ and no quantum operation touches the spectator.

This violates Assumption . It is the capacity leak identified in VII-E [6, Example ex:capacity].

Remark (Selective spectator states). For an entangled pair $(|00\rangle + |11\rangle)/\sqrt{2}$, local projective readout leaves the remote marginal $I/2$ nonselectively, but its conditional states are ketbra 00 and ketbra 11. Thus (iv) is not a claim that the spectator state is unchanged on each outcome. Nor may a subsequent controller inspect an otherwise excluded spectator record without changing the hypothesis.

Implementation equivalence and represented readout

Selective adjoint comparison

Let $\Psi_{h|g}$ and $\widetilde{\Psi}_{h|g}$ be two implementations with the same input and output label dictionaries.

Proposition (Exact implementation criterion). *The implementations agree on* $\mathscr{O}_{\mathrm{out}} = \mathrm{bigoplus}_h \mathscr{B}_h$ *for every hybrid input if and only if*

$$\Psi^*_{h|g}(B) = \widetilde{\Psi}^*_{h|g}(B) \qquad (B \in \mathscr{B}_h,\ g, h).$$

The equality also holds on $\mathscr{O}_{\mathrm{out}}$otimesL$(N)$ *for every untouched reference N, using the same extended input in both implementations.*

Proof. Choose an input supported in block g and an output observable supported in block h. Equality of all state expectations is equality of the Hermitian adjoint images. Complex linearity gives . Conversely, summing these identities proves output agreement.

For a reference observable $B \otimes C$, the two adjoint images are equal by tensoring with C. Such tensors span the reference-extended observation space.

Corollary (Implementation agreement through histories). *Suppose both implementations satisfy the same branchwise inclusions , and their branch adjoints agree on each* $\mathscr{O}_{t,h}$*. Starting from the same input, their complete histories have equal probabilities and equal accessible history-conditioned residual expectations.*

Proof. In the backward expectation formula from Theorem , replace the final adjoint by the other implementation's adjoint. The inclusion keeps its image in the previous observation space, where the next replacement is valid. Continue backwards to the initial state.

Equality of effects is only the case $B = I$. It cannot replace the selective-map criterion.

An exact dilation and its resource meaning

Fix an input label g. Choose a Kraus representation for every nonzero branch and write $a = (h, \alpha)$. Define

$$V_g \psi = \sum_{h,\alpha} |h, \alpha\rangle_A \otimes K_{h|g,\alpha} \psi.$$

Completeness implies $V_g^\dagger V_g = I$.

Destructive computational readout of *A*, followed by reporting only *h*, gives

$$\sum_h \mathrm{ketbra}\, hh_R \otimes \sum_\alpha K_{h|g,\alpha} X K_{h|g,\alpha}^\dagger.$$

The matter register is returned. The carrier is consumed. This is a represented instrument, not destructive readout of the matter source itself.

The Born readout interface is the specified semantic primitive of the Enter calculus [1]. No measurement-strategy uniqueness theorem is used as a premise here.

Proposition (One-shot environment ledger). *Let*

$$r_{h|g} = \mathrm{rank}\, J(\Psi_{h|g}), \qquad R_g = \sum_h r_{h|g},$$

where J is the unnormalized Choi matrix. Among dilations with orthogonal pointer sectors for the reported outcomes h, the minimum environment dimension is R_g.

A common pointer reserve of

$$e = \lceil \log_2 \max_g R_g \rceil$$

qubits suffices. If matter is encoded in s qubits, a unitary extension of uses those s matter qubits and e fresh pointer qubits. Full fine readout produces at most e classical bits per step.

Proof. A Choi matrix of rank $r_{h|g}$ has a decomposition into precisely that many rank-one positive terms, giving a Kraus family of that size. Conversely, a family with *q* operators gives a Choi sum of *q* rank-one terms, so $q \geq r_{h|g}$.

An environment sector of dimension q_h gives at most q_h Kraus operators when expanded in a sector basis. Thus $q_h \geq r_{h|g}$, and summing sectors gives the lower bound R_g. Minimal Kraus families in attain it.

Encode the pointer in *e* qubits. The isometry on the subspace with pointer input $|0\rangle^{\otimes e}$ extends to a unitary by completing orthonormal bases. Destructive readout consumes the pointer and returns a finite classical fine index.

This is an existence statement for a unitary dilation. It is not an exact-synthesis claim for arbitrary matrix entries in a fixed discrete gate library. After readout, any retained fine index is classical implementation history. It must

either be explicitly available to the controller or remain outside its observation boundary.

For a horizon T, retaining all fine indices requires at most Te bits. Storing all geometry labels additionally requires at most $(T+1)\lceil \log_2 |\mathrm{G}| \rceil$ bits. These are sufficient reserves, not optimal adaptive-memory bounds. No spectator is charged against them.

Immediate and coherent implementations that agree

Suppose a source instrument has Kraus operators M_a, with $\sum_a M_a^\dagger M_a = I$, and its outcome selects a successor $\Gamma_a(g)$ and unitary $U_{\Gamma_a(g)}$.

The immediate implementation reads out the pointer of

$$V_g^{\mathrm{src}}\psi = \sum_a |a\rangle_A \otimes M_a\psi$$

and then applies $U_{\Gamma_a(g)}$. The coherent implementation first applies

$$C_g = \sum_a \mathrm{ketbra}\, aa_A \otimes U_{\Gamma_a(g)}$$

and then reads out A.

Proposition (Exact source–feedback readout equivalence). *Both implementations have selective maps*

$$X\mathrm{longmapsto}\, U_{\Gamma_a(g)} M_a X M_a^\dagger U_{\Gamma_a(g)}^\dagger.$$

They therefore agree on the full classical-record–matter output, including every entangled reference.

Proof. Projection onto pointer outcome a gives

$$(\langle a|_A \otimes I) C_g V_g^{\mathrm{src}} = U_{\Gamma_a(g)} M_a.$$

This is the same branch operator as immediate readout followed by feedback. Equality of branch maps, and then tensoring with a reference, proves the conclusion.

Before readout, the coherent pointer is quantum, not an already existing classical geometry. The proposition compares the completed outputs. It does not equate arbitrary intermediate geometric traces or authorize an

intervening controller to inspect pointer coherences. This is consistent with the restricted deferred-readout boundary of VII-E and with precise circuit formulations of deferred measurement [6, 15, Theorem thm:deferred].

A counterexample using the same effects

Example (Equal records, different future geometry). For a qubit, consider two fair two-outcome instruments:

$$\Psi_a(X) = \tfrac{1}{2}X, \qquad \widetilde{\Psi}_a(X) = \tfrac{1}{2}\Delta_Z(X), \qquad a \in \{0,1\},$$

where

$$\Delta_Z(X) = P_0XP_0 + P_1XP_1.$$

Both have effects $I/2$. On input ketbra $+\,+$, their normalized residuals are respectively ketbra $+\,+$ and $I/2$, at trace distance 1/2.

If the next geometric output records an X-basis measurement, the first implementation gives

$$p(a,+) = \tfrac{1}{2}, \qquad p(a,-) = 0,$$

whereas the second gives $p(a,\pm) = 1/4$. The complete two-step geometric laws have total-variation distance 1/2.

The continuation observes a residual direction not determined by the first record effects. Its permission must be reflected in the backward observation space. This is the same record–residual separation emphasized by VII-E [6, Theorem thm:residual-obstruction and Theorem thm:luders]. Identical accessible geometry does not certify quantum recovery.

Two limitations of informative classical feedback

Deterministic packet selection and affinity

Proposition (Affine obstruction to nonconstant definite packets). *Let $\mathscr{S}$ be a convex set of quantum states. Suppose an affine map assigns to every $\rho \in \mathscr{S}$ a probability distribution concentrated at one classical label:*

$$T(\rho) = \delta_{\gamma(\rho)}.$$

Then γ is constant on $\mathscr{S}$.

The same conclusion holds for packets in any classical outcome space in which distinct packets can be separated by a measurable event.

Proof. If $\gamma(\rho) \neq \gamma(\sigma)$, then for $0 < t < 1$, affinity gives

$$T(t\rho + (1 - t)\sigma) = t\delta_{\gamma(\rho)} + (1 - t)\delta_{\gamma(\sigma)}.$$

This distribution is not concentrated at one label. The mixture belongs to $\mathscr{S}$, giving a contradiction. In a general outcome space, an event separating the two packets has probability strictly between zero and one on that mixture, with the same contradiction.

Applied to , this obstruction requires that the selected packet genuinely vary: $\theta\kappa \neq 0$, the source functional is nonconstant on the convex domain, and the packet's declared deficit distinguishes the outputs. It does not prohibit deterministic evolution of an already classical label.

Stochastic output, a nonconvex promised input class, or a controlled approximation is a different target. A nonlinear rule may also be retained explicitly outside the present channel programme.

Identity decompositions

Lemma (Completely positive branches of the identity). *If completely positive maps satisfy*

$$\sum_a \Psi_a = \mathrm{id}_{L(\mathscr{H})},$$

then

$$\Psi_a = p_a \mathrm{id}, \qquad p_a \geq 0, \quad \sum_a p_a = 1.$$

In particular, all record probabilities are state independent.

Proof. The Choi matrix of the identity is

$$J(\mathrm{id}) = |\Omega\rangle\langle\Omega|, \qquad |\Omega\rangle = \sum_{i=1}^{d} |i\rangle|i\rangle,$$

and has rank one. Each positive $J(\Psi_a)$ lies below $J(\mathrm{id})$, so its range is contained in the one-dimensional range of $J(\mathrm{id})$. Thus $J(\Psi_a) = p_a J(\mathrm{id})$. Choi injectivity gives the result.

Thus an informative instrument cannot retain the matter-unchanged property of on every quantum state.

Exact recovery and a constant complement

The next criterion is standard. It is also recorded in En02-C, but is restated and proved for the present use rather than imported with that manuscript's radiation benchmark [10, Lemma lem:decouple]. Channel-level continuity versions belong to the information–disturbance literature [14].

Theorem (Exact recovery criterion). *A finite-dimensional channel $\mathcal{N}$ has a CPTP left inverse if and only if a complementary channel has the form*

$$\mathcal{N}^{c}(X) = \mathrm{Tr}\,(X)\tau$$

for a fixed density operator τ.

Proof. Let $V : \mathscr{H}_{\mathrm{in}} \to \mathscr{H}_{\mathrm{out}} \otimes E$ be a Stinespring isometry.

Suppose a recovery exists and let $W : \mathscr{H}_{\mathrm{out}} \to \mathscr{H}_{\mathrm{in}} \otimes F$ dilate it. Then $(W \otimes I_E)V$ dilates the identity channel. Every Kraus operator of the identity is proportional to the identity by the rank-one Choi argument. Consequently this composite isometry has the form

$$\psi \mathrm{longmapsto}\, \psi \otimes \chi_{FE}$$

for one fixed unit vector. Tracing over the recovered input and F shows that the original complementary output is constant.

Conversely, diagonalize $\tau = \sum_j \lambda_j \mathrm{ketbra}\, jj$, omitting zero eigenvalues. Write $V = \sum_j V_j^{\mathrm{raw}} \otimes |j\rangle$. The complementary identity implies

$$(V_k^{\mathrm{raw}})^{\dagger} V_j^{\mathrm{raw}} = \delta_{jk} \lambda_j I.$$

Hence $V_j^{\mathrm{raw}} = \sqrt{\lambda}_j V_j$, where the V_j are isometries with mutually orthogonal ranges. Let $P = \sum_j V_j V_j^\dagger$, and choose a density operator ω_* on the input. Define

$$\mathscr{R}(Y) = \sum_j V_j^\dagger Y V_j + \mathrm{Tr}\,((I - P)Y)\omega_*.$$

This is completely positive and trace preserving. Since $\mathscr{N}(X) = \sum_j \lambda_j V_j X V_j^\dagger$, orthogonality gives $\mathscr{R}\mathscr{N}(X) = X$.

Corollary (An informative record prevents universal recovery). *Let $\{\Psi_a\}$ be an instrument. If at least one effect $\Psi_a^*(I)$ is not a scalar multiple of I, then:*

1. *its nonselective matter channel $\sum_a \Psi_a$ has no CPTP left inverse;*

2. *its complete classical-record–matter channel $\mathscr{I}(X) = \mathrm{bigoplus}_a \Psi_a(X)$ also has no CPTP left inverse. Both assertions concern recovery of every input state.*

Proof. For (i), use the dilation with environment basis $|a, \alpha\rangle$. Measuring its environment gives the instrument probabilities. A constant complement would make every such probability independent of the input, contradicting the nonscalar effect. Apply Theorem .

For (ii), suppose $\mathscr{R}\mathscr{I} = \mathrm{id}$. Restrict the recovery to each classical output block, obtaining a CPTP map $\mathscr{R}_a$. The completely positive maps $\mathscr{R}_a\Psi_a$ sum to the identity, so Lemma gives $\mathscr{R}_a\Psi_a = p_a\mathrm{id}$. Taking traces and using trace preservation of $\mathscr{R}_a$ implies $\Psi_a^*(I) = p_a I$, again a contradiction.

A nondemolition measurement of one observable is compatible with this corollary: its eigenstates may be preserved while coherence between eigenspaces is disturbed. Recovery on a proper code subspace is also a different task.

Proposition (Constant-qubit calibration). *For a constant qubit channel $\mathscr{N}(X) = \mathrm{Tr}\,(X)\tau$,*

$$\inf_{\mathscr{R}\ \mathrm{CPTP}} d_\diamond(\mathscr{R}\mathscr{N}, \mathrm{id}_2) = \frac{3}{4}.$$

Proof. Every decoded channel is constant, with some output σ. On a maximally entangled input its output is $I/2 \otimes \sigma$, whose Bell overlap is 1/4. Testing the Bell projector gives half-diamond error at least 3/4.

Preparing $I/2$ attains the bound. Indeed, the completely depolarizing qubit channel equals the uniform mixture of the four Pauli conjugations. Subtracting the identity and using the triangle inequality gives half-diamond distance at most 3/4.

Only this constant-channel calibration is used. No streaming-memory or radiation error from a different channel is transferred.

Balance laws and admissibility without postselection

Operator tests for energy

Let a declared Hermitian hybrid observable be

$$\mathscr{E}_g = E_{\mathrm{geom}}(g)I + H_{\mathrm{m}}(g) + E_{\mathrm{res}}(g)I.$$

The reservoir term, if used, is part of the label specification. The evolution parameter and physical interpretation must be declared independently of a circuit tick or slicing parameter.

Proposition (Exact expectation balance). *Expected energy is conserved for every hybrid input if and only if*

$$\sum_h \Psi^*_{h|g}(\mathscr{E}_h) = \mathscr{E}_g \qquad (g \in \mathrm{G}).$$

The branchwise relations

$$\mathscr{E}_h K_{h|g,\alpha} = K_{h|g,\alpha}\mathscr{E}_g$$

are sufficient.

If the norm of the defect in is at most δ for every g, the expected one-step error is at most δ, and the accumulated error after n steps is at most nδ for a common observable and uniform defect.

Proof. Expected conservation is $\langle \mathscr{E}, \mathfrak{F}\rho\rangle = \langle \mathscr{E}, \rho\rangle$. Testing all states supported in one input block gives the operator identity.

Under ,

$$\sum_{h,\alpha} K^{\dagger}_{h|g,\alpha} \mathscr{E}_h K_{h|g,\alpha} = \sum_{h,\alpha} K^{\dagger}_{h|g,\alpha} K_{h|g,\alpha} \mathscr{E}_g = \mathscr{E}_g.$$

For a defect D_g,

$$|\sum_g \operatorname{Tr}(D_g \rho_g)| \le \max_g \|D_g\|_\infty \sum_g \operatorname{Tr} \rho_g.$$

Iteration and the triangle inequality give the accumulated bound.

Condition is spectral intertwining. It is not the assertion that postselecting an energy-correlated outcome preserves the unconditional input mean on that branch.

A separate resource ledger

Let $r(g) \ge 0$ be a resource budget and let $c_{h|g} \ge 0$ be a declared edge cost satisfying

$$r(h) + c_{h|g} = r(g) \quad \text{on every nonzero branch.}$$

Proposition (Resource balance). *On every supported history,*

$$r(g_n) + \sum_{t=1}^{n} c_{g_t|g_{t-1}} = r(g_0).$$

Its expectation satisfies

$$(\mathfrak{F}^* r)_g = r(g) I - \sum_h c_{h|g} E_{h|g}.$$

Proof. The history identity telescopes . For the expected identity,

$$(\mathfrak{F}^* r)_g = \sum_h r(h) E_{h|g} = \sum_h (r(g) - c_{h|g}) E_{h|g},$$

and the effects sum to I.

These costs may count permissions, attempts, records, or other declared finite resources. They are not automatically physical energies.

Complete fallback design

Proposition (Admissible rerouting). *Suppose* $\{\Phi_{a|g}\}_a$ *is a complete instrument and* $a \mapsto \Gamma_a(g)$ *is a fixed routing*

table whose outputs all have valid certificates, including any declared no-update output. Then

$$\Psi_{h|g} = \sum_{a\,:\,\Gamma_a(g)\,=\,h} \Phi_{a|g}$$

is a complete hybrid transition.

Observation compatibility holds if each routed map satisfies

$$\Phi_{a|g}^{*}(\mathcal{O}_{\Gamma_a(g)}) \subseteq \mathcal{O}_g.$$

Proof. Grouping completely positive maps preserves complete positivity, and summing all groups gives the original trace-preserving instrument. Summing the indicated adjoint inclusions gives the branch inclusions.

Rerouting does not undo the matter disturbance of the rejected proposal. Nor does rerouting automatically preserve an observation boundary established for a different output algebra.

Example (Deleting a branch is not a channel repair). If the only retained branch is $\Phi(X) = P_0 X P_0$, its trace is $\mathrm{Tr}\,(P_0 X)$, not $\mathrm{Tr}\,X$. The normalized successful output

$$\rho \mathrm{longmapsto} \frac{P_0 \rho P_0}{\mathrm{Tr}\,(P_0 \rho)}$$

is not a trace-preserving linear map on the full input domain. A complete model must retain an admissible failure branch or another complete fallback.

Operational locality with explicit controller support

Nonselective remote invariance

Proposition (Untouched remote reduction). *For a local instrument $\{\Psi_a^R\}$ on R, with $\sum_a \Psi_a^R$ trace preserving,*

$$\mathrm{Tr}_R \left[\sum_a (\Psi_a^R \otimes \mathrm{id}_{R^c})(\rho) \right] = \mathrm{Tr}_R \rho.$$

This holds for entangled inputs. It does not assert equality of conditional remote states.

Proof. Pair the remote output with B. Its expectation is

$$\operatorname{Tr}\left(\left[\sum_a \Psi_a^{R*}(I_R)\right] \otimes B\ \rho\right) = \operatorname{Tr}\left((I_R \otimes B)\rho\right).$$

Feedback compiled into a local hybrid network

Let a finite site set X carry local algebras

$$\mathfrak{X}_x = \operatorname{bigoplus}_{c_x} L(\mathscr{H}_x).$$

The local classical variables include all controller records read at that site. The global algebra is $\operatorname{bigotimes}_{x \in X} \mathfrak{X}_x$, equivalently a direct sum over classical configurations.

At instruction t, suppose a fixed envelope $R_t \subseteq X$ contains every quantum and classical register read or written by that instruction, and the complete instruction is a channel

$$\Phi_t^{R_t} \otimes \operatorname{id}_{X \setminus R_t}.$$

Within R_t, the channel may select different operations from its classical controller blocks. The envelope must nevertheless include those controls.

For $Y \subseteq X$, define backwards

$$S_T = Y, \qquad S_{t-1} = \begin{cases} S_t \cup R_t, & R_t \cap S_t \neq \varnothing, \\ S_t, & R_t \cap S_t = \varnothing. \end{cases}$$

Theorem (Hybrid-controller support bound). *Every output observable supported on Y pulls back to an input observable supported on S_0. Equal input reductions on S_0 therefore give equal output reductions on Y.*

If X has a metric and $\operatorname{diam} R_t \leq \Delta$*, then*

$$S_0 \subseteq \{x : d(x, Y) \leq T\Delta\}.$$

Proof. If R_t is disjoint from the current support, the observable is the identity on that entire envelope. The channel adjoint is unital, so the envelope contributes no support. If they meet, support can enlarge only to their union. Induction proves .

A new point lies within distance Δ of a point in the previous backwards set. The triangle inequality gives the

metric bound. Finally, equality of input reductions gives equality of every pulled-back output expectation.

This theorem does not inherit locality from a controller that reads a global geometry label and then selects a remote support. Such a controller must itself be included in the envelopes.

The distinction is the one already present in VII-E's prescribed schedule theorem and VIII-C's prescribed block-channel bound [6, Theorem thm:locality and Proposition prop:direction], [9, Theorem thm:C5]. The present extension permits feedback inside a declared local hybrid channel. It is still a dependency theorem, not a spacetime light-cone theorem. No physical directional causal dominance is deduced.

Assisted stability and finite Choi certificates

A classical-input diamond norm

For a hybrid linear map D, define its diamond norm through the canonical full-matrix extension that first pinches the input label and then applies the direct-sum map. Equivalently, one may amplify the direct-sum map by arbitrary matrix references and use the trace norm .

For two transitions, set

$$D_{h|g} = \Psi_{h|g} - \widetilde{\Psi}_{h|g}, \qquad D_g(X) = \operatorname{bigoplus}_h D_{h|g}(X).$$

Proposition (Exact column reduction). *The assisted hybrid norm satisfies*

$$\|\mathfrak{F} - \widetilde{\mathfrak{F}}\|_\diamond = \max_g \|D_g\|_\diamond.$$

Moreover,

$$\|D_g\|_\diamond \le \sum_h \|D_{h|g}\|_\diamond.$$

Proof. For an amplified block input $X = (X_g)_g$,

$$\|[(\mathfrak{F} - \widetilde{\mathfrak{F}}) \otimes \mathrm{id}](X)\|_1 \le \sum_g \|(D_g \otimes \mathrm{id})(X_g)\|_1 \le \max_g \|D_g\|_\diamond \sum_g \|X_g\|_1.$$

For the full-matrix extension, pinching is a trace-norm contraction also after amplification. This follows either

from duality with a unital completely positive adjoint or by averaging diagonal phase unitaries. Hence the same upper bound holds there.

An input supported in one maximizing block gives the reverse inequality. The branch bound is the triangle inequality in the output direct sum.

Choi upper and lower bounds

For an arbitrary linear map $D : L(\mathbb{C}^d)\text{toL}(K)$, write

$$J(D) = \sum_{i,j=1}^{d} \text{ketbra}\, ij \otimes D(\text{ketbra}\, ij), \qquad \chi(D) = J(D)/d.$$

Lemma (Finite Choi certificate). *For every such map,*

$$\|\chi(D)\|_1 \le \|D\|_\diamond \le d\|\chi(D)\|_1.$$

For a column map D_g, its Choi matrix is the direct sum of the branch Choi matrices in the output label.

Proof. Apply D to one half of the normalized maximally entangled state. Its output is $\chi(D)$, giving the lower bound.

For the upper bound, adjoin an arbitrary reference space N. Any unit vector in $N \otimes \mathbb{C}^d$ can be written

$$u = (A \otimes I)|\Omega\rangle, \qquad \text{Tr}\,(A^\dagger A) = 1,$$

with $A : \mathbb{C}^d \to N$, and therefore $\|A\|_\infty \le 1$. For two unit vectors u, v, represented by A, B,

$$(\text{id}_N \otimes D)(\text{ketbra}\, uv) = (A \otimes I)J(D)(B^\dagger \otimes I).$$

Its trace norm is at most $\|J(D)\|_1$. A singular-value decomposition of an arbitrary input operator extends the bound to

$$\|(\text{id}_N \otimes D)(X)\|_1 \le \|J(D)\|_1 \|X\|_1.$$

Taking the supremum proves the upper bound. The direct-sum assertion follows from linearity of the Choi

construction.

These are certification bounds, not numerical results inherited from another finite channel. They use the normalization explicitly.

Accumulation for terminal states and full histories

Theorem (Finite-execution stability). *Suppose two executions use the same label dictionaries and satisfy*

$$\|\mathfrak{F}_t - \widetilde{\mathfrak{F}}_t\|_\diamond \le \varepsilon_t.$$

Their terminal channels obey

$$\|\mathfrak{F}_T \cdots \mathfrak{F}_1 - \widetilde{\mathfrak{F}}_T \cdots \widetilde{\mathfrak{F}}_1\|_\diamond \le \sum_{t=1}^{T} \varepsilon_t.$$

The same bound holds for their complete geometry-history channels.

For a common input, including an arbitrary reference, the trace distance of the corresponding outputs, and the total-variation distance of their geometric history laws, are at most

$$\min\left\{1, \frac{1}{2}\sum_{t=1}^{T} \varepsilon_t\right\}.$$

Proof. Insert and subtract one implemented step at a time. Each term has the difference of one pair of steps sandwiched between channels. Channels have diamond norm one, and the diamond norm is submultiplicative [13]. The triangle inequality gives .

For history retention, a current history is a classical input label. Its next-step column is D_{g_t}, with the preceding history copied into the output label. Copying that fixed classical prefix does not change the norm of the column. Proposition therefore gives the same one-step bound, now on the history algebra. Apply the same telescoping argument.

A common normalized input gives trace-norm difference at most the channel diamond bound. Trace distance contributes the factor 1/2. Reading only the geometric record is a channel, so it cannot increase trace distance; on

diagonal states that distance is total variation. Distances between states are at most one.

Restricted unassisted comparison

For an observation space $\mathscr{O}$, define

$$d_{\mathscr{O}}(\mathfrak{F}, \widetilde{\mathfrak{F}}) = \frac{1}{2}\sup_{\rho} \sup_{\text{substack } B = B^{\dagger} \in \mathscr{O}}$$

Then

$$d_{\mathscr{O}}(\mathfrak{F}, \widetilde{\mathfrak{F}}) = \frac{1}{2} \sup_{\text{substack } B = B^{\dagger} \in \mathscr{O}}$$

Indeed, interchange the suprema and optimize the Hermitian expectation by an eigenstate in a maximizing block. This is the hybrid form of VII-E's accessible-output distance [6, Proposition prop:accessible]. It is unassisted unless a reference is included explicitly.

Guard stability is a separate obligation

An exact clock guard $j = 0$ is stable under an exact finite representation, but not under arbitrary perturbation of a real parameter. Similarly, if a length is $c|s|$, a floor report at resolution h is zero whenever $0 < c|s| < h$. It cannot serve as an exact event detector.

Theorem applies to already specified channels on the same finite dictionaries. It does not convert a discontinuous geometric guard into a uniformly continuous map of real geometric data. Such a claim requires a separate margin or event-resolution theorem, as emphasized in VII-E and VII-F [6, 7].

An explicit resource-budgeted feedback witness

This section witnesses the consistency of the finite specification. It is not a replacement for VII-F's particle-link incidence construction.

A finite certified geometric family

Fix an integer budget $B \geq 1$, a tick duration $\tau > 0$, and $0 < \delta < \tau$. Set

$$\mathrm{G}_B = \{(k,q) : 0 \le k \le B,\ q \in \mathrm{Z},\ |q| \le k, ?q \equiv k \bmod 2\}.$$

The resource is $r(k,q) = B - k$. The controller tick is $k\tau$. The selected slice height is

$$T(k,q) = k\tau + \delta q.$$

These two quantities are distinct.

Use the fixed ultrastatic spinless cone

$$M = \mathbb{R}_T \times C_{\pi/2}, \qquad ds^2 = -dT^2 + dr^2 + r^2 d\theta^2, \qquad 0 \le \theta \le 3\pi/2,$$

with its wedge boundary rays identified. Each label denotes the horizontal slice $T = T(k,q)$, with the same fixed sector cellulation, stationary axis, and any fixed annular collar.

For $k < B$, define

$$\Gamma_a(k,q) = (k+1, q+a), \qquad a \in \{-1, +1\}.$$

At $k = B$, the label is absorbing.

Lemma (Complete branch certificate). *Every successor in belongs to* G_B. *Its slice is embedded and spacelike, and advances in spacetime time by* $\tau + a\delta > 0$.

The complete affine peripheral word is

$$J = (R(\pi/2), 0)$$

on every label, with the same marked stationary axis. Location transport is the identity on the persistent cell labels. The resource decreases by exactly one on every active edge.

Proof. The inequalities and parity in are preserved by $k \mapsto k+1$, $q \mapsto q+a$. A horizontal graph has induced metric equal to the spatial cone metric and is embedded. Formula gives the time increment.

The upper seam maps to the lower seam by the rotation $R(\pi/2)$, which fixes the time axis pointwise. A positively oriented primitive meridian crosses that seam once; its angular lift advances by $3\pi/2$. Its affine word is therefore J.

Moving the adapted horizontal chart by $-T(k,q)e_0$ commutes with J, so there is no induced affine time translation. The cellulation and collar are unchanged spatially, giving identity cell-label transport. Finally,

$B-(k+1)=B-k-1$.

The cone interpretation and the need to retain full affine data are consistent with the classical point-particle description and the fixed-cone certificate of VII-F [22, 7]. Here no deficit or worldline changes.

The unsharp source instrument

Let $\mathscr{H}=\mathbb{C}^2$, P_j = ketbra jj, and

$$Q=P_1-P_0.$$

Choose $0\le\eta\le 1$ and define

$$E_a=\tfrac{1}{2}(I+a\eta Q),\qquad M_a=\sqrt{E_a}.$$

Let

$$U_{k,q}=\exp(-i\theta_{k,q}Q),\qquad \theta_{k,q}=\pi q/8.$$

For an active input $g=(k,q)$, set

$$\Psi_{\Gamma_a(g)|g}(X)=U_{\Gamma_a(g)}M_aXM_aU_{\Gamma_a(g)}^{\dagger},$$

with all other branches zero. At $k=B$, use $\Psi_{g|g}=\mathrm{id}$.

This is an operational measurement–feedback prescription. Newtonian measurement-feedback models provide related motivation, not a gravitational validation of this finite construction [20].

Proposition (Exact source and disturbance formulas). *The example is a complete hybrid channel. For a normalized active matter state ρ,*

$$p_a=\tfrac{1}{2}(1+a\eta\,\mathrm{Tr}\,(Q\rho)),$$

$$p_+-p_-=\eta\,\mathrm{Tr}\,(Q\rho).$$

Before the geometry-dependent unitary, its nonselective source map

$$\mathscr{M}_{\eta}(X) = M_{+}XM_{+} + M_{-}XM_{-}$$

preserves diagonal blocks and satisfies

$$P_0\mathscr{M}_{\eta}(X)P_1 = \sqrt{1-\eta^2}\,P_0XP_1.$$

Writing $x = \delta q$, the active geometric increment has

$$\mathbb{E}[\Delta x \mid \rho] = \delta\eta\,\mathrm{Tr}\,(Q\rho),$$

$$\mathrm{Var}\,(\Delta x \mid \rho) = \delta^2(1-\eta^2\mathrm{Tr}\,(Q\rho)^2).$$

Proof. The positive effects sum to I, so $\sum_a M_a^{\dagger}M_a = I$. Unitary feedback does not change branch traces, proving and completeness.

On P_0, the two square-root eigenvalues are $\sqrt{(1-\eta)/2}$ and $\sqrt{(1+\eta)/2}$, in opposite order to their values on P_1. Their two cross products sum to $\sqrt{1-\eta^2}$; their squared values sum to one on each sector. This proves . Finally, $\Delta x = \delta a$, so its mean follows from and its second moment is δ^2.

These are model-specific formulas, not continuous decoherence–diffusion inequalities. The latter require their own generator hypotheses [19].

Observation, feedback, and implementation

Choose the source observation algebra

$$\mathscr{A} = \mathrm{span}_{\mathbb{C}}\,\{P_0, P_1\}.$$

Both M_a and $U_{k,q}$ commute with this algebra. Therefore

$$\Psi_{h|g}^{*}(\mathscr{A}) \subseteq \mathscr{A}$$

on every branch, including absorbing ones. Theorems and apply to every input and every entangled spectator

extension.

For $0 < \eta < 1$, the two feedback unitaries at the successors of one active label act differently on surviving off-diagonal matter blocks. Thus geometry affects the subsequent matter channel. That phase dependence is not observable on $\mathscr{A}$, which records only populations. A continuation measuring coherence must declare an enlarged observation boundary; it is not licensed to treat coherence as both inaccessible and later observable.

The source dilation

$$\psi \mathrm{longmapsto}\, |+\rangle_A M_+ \psi + |-\rangle_A M_- \psi$$

uses one fresh pointer qubit. Immediate readout followed by , and coherent controlled feedback followed by readout, agree as full transition instruments by Proposition . Their complete residual outputs, not merely their geometric probabilities, coincide.

The classical label update is a finite table and has a reversible completion with retained input history: an injective map $z \mapsto (f(z), z)$ can be extended to a permutation on a finite enlarged code space by exchanging disjoint input–output pairs. This does not claim discrete-gate exact synthesis of the unsharp quantum rotations for arbitrary η.

Resource accounting

Every active branch consumes one permission, regardless of the outcome. No measurement is attempted at $k = B$. Consequently,

$$(\mathfrak{F}^* r)_{k,q} = \begin{cases} (B-k-1)I, & k < B, \\ 0, & k = B. \end{cases}$$

On an execution initially at k_0, the number of source attempts after n ticks is exactly $\min\{n, B - k_0\}$.

One fresh pointer qubit suffices per active attempt. Retaining its readout consumes one additional classical history bit. The matter qubit is returned; the pointer is destructively consumed. Spectator storage is separate. These are resource statements, not an energy model for the measurement apparatus.

An exact classical sector description

Let $\lambda_j = 2j - 1$, so $QP_j = \lambda_j P_j$. The branches satisfy

$$\Psi_{\Gamma_a(g)|g}(P_j) = p_a(j)P_j, \qquad p_a(j) = \tfrac{1}{2}(1 + a\eta\lambda_j).$$

Thus, on states diagonal in the source basis, the full evolution is exactly the classical Markov chain on (g, j) with fixed j and transition probabilities $p_a(j)$.

For an arbitrary initial qubit state at one label, its geometric history law is the mixture of those two classical sector laws with weights ρ_{00}, ρ_{11}. Off-diagonal blocks have zero trace at every history and do not contribute. Outcomes are independent conditional on j, not generally independent after mixing the sectors.

Proposition (Quantitative approximation to the diagonal model). *Start at one label $g_0 = (k_0, q_0)$. Let $\Delta_Z\rho = P_0\rho P_0 + P_1\rho P_1$, and compare the complete history–matter outputs from ρ and $\Delta_Z\rho$. After $n \le B - k_0$ active steps,*

$$d_{\mathrm{tr}}(\mathcal{H}_n(\rho), \mathcal{H}_n(\Delta_Z\rho)) = |\rho_{01}|(1 - \eta^2)^{n/2}.$$

Their geometric history laws agree exactly. Forgetting histories can only decrease the distance in .

Proof. On a fixed history, the difference has only a pair of conjugate off-diagonal entries. Its trace distance contribution is the absolute value of one entry. The diagonal feedback unitaries change only its phase.

For either outcome,

$$\langle 0|M_a|0\rangle\langle 1|M_a|1\rangle = \tfrac{1}{2}\sqrt{1 - \eta^2}.$$

Each length-n history therefore contributes $|\rho_{01}|(\sqrt{1 - \eta^2}/2)^n$. There are 2^n histories, so their direct-sum contributions give . Equal diagonals give equal history probabilities by . Marginalization over histories is a channel.

This is a controlled approximation to a named finite classical model. It is not an $\hbar \to 0$ limit, a continuum limit, or

an Einstein limit. At $\eta = 0$, the source learns nothing and no dephasing occurs. At $\eta = 1$, one active measurement removes all source coherence, while its sector eigenstates remain unchanged.

An explicit interior parameter bound

Proposition (Pilot implementation stability). *Compare two active instruments with parameters* $\eta, \widetilde{\eta} \in [0, \eta_*]$, *where* $\eta_* < 1$, *and feedback phases* $\theta_h, \widetilde{\theta}_h$. *For the same routing table,*

$$\|\mathfrak{F} - \widetilde{\mathfrak{F}}\|_\diamond \le 4\left[\frac{|\eta - \widetilde{\eta}|}{2\sqrt{2(1-\eta_*)}} + \max_h |\theta_h - \widetilde{\theta}_h|\right].$$

The bound may be capped by 2.

Proof. The derivatives of the scalar entries of M_a have magnitude at most $1/(2\sqrt{2(1-\eta_*)})$. Hence the first term bounds $\|M_a - \widetilde{M}_a\|_\infty$. Also

$$\|e^{-i\theta Q} - e^{-i\widetilde{\theta}Q}\|_\infty \le |\theta - \widetilde{\theta}|.$$

The branch operators $K_a = U_a M_a$ are contractions, and

$$\|\mathrm{Ad}_{K_a} - \mathrm{Ad}_{\widetilde{K}_a}\|_\diamond \le (\|K_a\|_\infty + \|\widetilde{K}_a\|_\infty)\|K_a - \widetilde{K}_a\|_\infty.$$

This follows by expanding $K_a X K_a^\dagger - \widetilde{K}_a X \widetilde{K}_a^\dagger$ into two terms, also after amplification. Summing the two branch bounds and using Proposition gives . Two channels have diamond distance at most 2.

A uniform Lipschitz estimate in the parameter η is not asserted at $\eta = 1$: the square-root coherence factor itself is not Lipschitz there. Channel-norm certification remains meaningful at that endpoint.

Theorem (Finite observation-compatible feedback model). *The model – has:*

1. *a finite set of completely specified, admissible geometric and controller labels;*

2. *a fixed qubit matter space and state-independent completely positive trace-preserving hybrid evolution;*

3. *matter-dependent successor probabilities determined by the declared source algebra;*

4. *an admissibility certificate for every nonzero branch, without postselection;*

5. *branchwise compositional observation compatibility;*

6. *exact spectator consistency for entangled extensions and complete histories;*

7. *two full-instrument-equivalent readout implementations;*

8. *an exact permission balance and finite pointer/history resource reserve;*

9. *assisted implementation stability and a quantitative approximation to the declared diagonal classical model.*

For $0 < \eta < 1$*, both source sensitivity and geometry-dependent matter phase operations are nontrivial. The ambient cone, particle worldline, deficit, and peripheral affine holonomy remain fixed.*

Proof. Lemma proves (i) and (iv). Theorem and Proposition prove (ii) and (iii). Commutation with gives the branch inclusions, proving (v) by Theorem . The specified controller and separate reserves satisfy Assumption ; Theorem gives (vi).

Proposition gives (vii). Equation and Proposition give (viii). Theorem , Lemma , Proposition , and Proposition give (ix). The final statements follow from the explicit source effects, distinct successor phases, and the fixed-cone certificate.

Geometric and field-theoretic boundaries

How a VII-F event may enter a later instrument

A later transition may first produce a classical source outcome and then select a certified geometric edge. The source instrument and the geometry-only event remain distinct operations.

For the VII-F incidence event, the exact guard is still $j = 0$, with incoming/outgoing germ and permission data included in the classical label. A source outcome may prepare a declared readiness register, but it may not replace the exact guard by a rounded length test. Applying Proposition requires an admissible fallback for every outcome, not deletion of an invalid event branch [7, Contract contract:G, Proposition prop:blind, and Theorem

thm:implementation].

Such an integration would change neither the deficit nor the worldline merely by invoking the finite channel theorem. VIII-B's supplied packet family and VIII-C's promised-token composition are comparison constructions, not a derived dynamical law [8, 9].

Two geometric negative tests

Example (A stale deficit identification). Suppose a cone realization has deficit μ, but a proposed output packet reports $\nu \neq \mu$ while retaining its seam word $(R(\mu), 0)$. That output does not match the reported deficit in the fixed framing. For planar rotations,

$$\|R(\mu) - R(\nu)\|_2 = 2|\sin((\mu - \nu)/2)|.$$

The mismatch is not repaired by changing a packet label.

Example (Correct Lorentz part, wrong affine word). For $\varepsilon \neq 0$,

$$J_\varepsilon(z) = R(\mu)z + \varepsilon e_0$$

has the same Lorentz part as the spinless identification, but no fixed point: its time coordinate increases by ε. In particular, it does not fix the stationary worldline pointwise. This is why a certificate must check the full affine peripheral word, not only its Lorentz factor [7].

Changing a deficit, moving an axis, or transferring physical energy between matter and geometry requires new matching and evolution proofs. The finite operator test is necessary for a declared energy balance but does not supply those geometric proofs.

Why VII-A–D do not provide a source instrument

The supplied manuscripts establish distinct results:

• VII-A gives a second-order classical plateau intertwiner, not a quantum source algebra [2].

• VII-B excludes the unchanged sharp-Cauchy insertion from its declared off-shell microcausal polynomial

domain; its linear on-shell repair is not an interacting source instrument [3, Theorem thm:main].

• VII-C excludes one normal-topology Wick extension under its stated continuity and old-product agreement requirements, not every possible boundary theory [4, Theorem thm:coefficient and Corollary cor:noalgebra].

• VII-D gives one strong numerical finite part and a complete regulator conversion, not an interacting observable algebra or stress tensor [5, Theorems thm:finitepart and thm:conversion]. Consequently,

$$\text{finite numerical graph coefficient} \nRightarrow \text{quantum source instrument}.$$

A field-theoretic successor must independently specify a source domain, state, renormalization, smearing, and completely positive instrument. Only then may normalization changes be tested against the geometric observation boundary. VII-E's descent and counterterm-blind tests provide a comparison dictionary, not a product construction [6, Propositions prop:descent, prop:blind-tests, and prop:presentation].

Nothing in the present finite argument constructs a missing characteristic graph product, a quantum Ward identity, or an admitted interacting coefficient.

What is left to a genuinely gravitational successor

A gravitational extension must declare:

1. the dynamical geometric variable and its physical clock;

2. the gravitational evolution and constraints;

3. complete affine transitions, axes, collars, and matching for every branch;

4. total energy and any apparatus or reservoir;

5. directional physical propagation, not merely scheduled support;

6. a controlled approximation to a specified gravitational model. A continuous-time or continuum limit is not obtained by the finite proofs above. Existing classical–quantum limiting constructions provide comparison frameworks, whose scaling assumptions must be matched rather than imported [21].

Conclusion

A finite matter–geometry feedback step can be a genuine channel only after its classical controller, quantum instrument, geometric certificates, and observation boundary have been declared together.

The unconditional law uses subnormalized blocks and fixed completely positive transition maps. Exact observation compatibility is an adjoint inclusion. For stochastic histories, it must hold branchwise; the smallest future-sensitive observation spaces can be constructed backwards and, in the homogeneous finite case, stabilize after finitely many enlargements. Entangled spectators preserve every unnormalized history block after restriction when the entire controller is blind to them.

Implementation independence is conditional and testable: selective maps must agree on the declared continuation observables. Equal effects alone are insufficient. Informative classical records also cannot coexist with exact preservation, or exact recovery, of every quantum input.

The explicit qubit model proves that these requirements, an exact resource ledger, and quantitative stability can be satisfied simultaneously. Its geometric branches are strictly ordered slices of one fixed cone. That is a closed finite operational result, not a theory of semiclassical gravity.

$$\text{boxed} \quad \begin{array}{c} \text{fixed CP transition instrument} \\ + \text{ complete classical controller} \\ + \text{ branchwise observation closure} \\ + \text{ declared geometric certificates} \end{array} \Rightarrow \text{?consistent finite feedback and history comparison.}$$

The gravitational dynamics and field-theoretic source stages remain separate constructions.

Book expansion notes (t433u)

A completely positive observation instrument is not a metric and not a backreaction law. This chapter pairs with VII-E [6]; the two chapters are not merged. Finite spacetime source record: [28].

Standing ledger. Not spacetime emergence; not Einstein dynamics; not a Page curve, island, bounce, vacuum-energy selection, or de Sitter state as an Enter theorem; not a derived metric. O.1 stays NOT_CLOSED.

XVI-N is not discharged. XV-B is not opened. No quantum coefficient is admitted. Host C is not created. Dom (T) remains undetermined; J2 remains unresolved. Problems 6, 7, and 9 are named, not solved. Gate 0 and ST4b stay NOT_SUPPLIED. Host-A stays NAMED_COMPARISON_ONLY $\{\mathrm{GI}, \mathrm{CB}\}$. The shredder is not Gibbons–Hawking. Cited sheets are source records, not premises. Conjectures stay conjectures. wired=false.

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