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Landmark VII-H Normalization-independent connected quantum responses: complete ambiguity, observable descent, and a nonzero free-field benchmark

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Abstract

We formulate prescription independence at the response level and complete the three-stage construction for one explicitly declared free-field benchmark. The source and readout are the vacuum-centred Wick square of a massless scalar on four-dimensional Minkowski spacetime. At first order in a compact external source, the complete connected vacuum susceptibility has a punctured retarded kernel $\hbar\Theta(t)\delta'(t^2 - |\mathbf{x}|^2)/(4\pi^3)$.

A locally integrable logarithm gives an explicit retarded extension. Reality, translation covariance, agreement away from coincidence, and scaling degree at most four imply that every admissible normalization differs by exactly $\hbar c\delta_0$. Thus the complete response ambiguity is determined, rather than inferred from two regulators. We construct an admissible annular test whose pairing is exactly $-\hbar t_0/(2\pi^2)$ for every prescription, and provide a single delayed source/readout experiment with a rigorously nonzero response.

We also prove the general observable-descent criterion, its test-determined topological separation condition, reconstruction descent, and a finite-bank certificate that remains valid when the underlying ambiguity is infinite-dimensional. Calibration, minimum-variance weights, regulator smoothing, and imperfect annihilation are treated separately.

The benchmark is not the reflected triangle of VII-D and is not the unresolved gravitational characteristic

coefficient. The latter construction remains open on the supplied data. Classical admission, the nonzero auxiliary remainder of XII-F, and finite instruments in VII-G are not substituted for that missing quantum response.

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par **Exact release** par **Constructed:** one connected quantum response at first source order; its complete ambiguity in a stated prescription class; an exactly evaluated nonzero invariant test; a delayed source/readout realization; and the general descent and stability results.

Explicit restriction: the affirmative result is for a local free massless scalar benchmark with fixed vacuum, calibrated field and composite normalization, and no physical spacetime boundary.

Not constructed: an embedding of VII-D's reflected triangle in a complete interacting response; the inherited gravitational characteristic quantum coefficient; an interacting gravitational Ward identity; or a measurement instrument for that coefficient.

The benchmark demonstrates the proposed logical sequence without claiming to close an unrelated analytic gate.

par

The comparison problem and the selected target

The question is not whether a preferred finite part can be assigned. It is whether a response, after physical matching, has a nonzero pairing independent of every permitted prescription change:

$$\text{boxed defined response} \;\to\; \text{complete response ambiguity} \;\to\; \text{nonzero invariant observation.}$$

Three distinctions govern the construction:

$$\begin{gathered}\text{one graph coefficient} \neq \text{complete response},\\ \text{a local numerical counterterm} \neq \text{its propagated response image},\\ \text{a quotient criterion} \neq \text{a nonzero quotient class}.\end{gathered}$$

Local renormalization gives extension and comparison theorems under specified hypotheses [1, 2, 3]. Finite-renormalization frameworks compare interactions and observables together [4, 5]. These results do not identify the ambiguity of an unspecified nonlocal response.

Why a benchmark is selected explicitly

The supplied corpus does not contain a complete construction of the proposed reflected-triangle or gravitational connected response. In particular, the supplied XIV-I and XIV-J statements stop before the required full characteristic composition.

We therefore do not announce a completion of that response. Instead, we select a different model for which all three arrows in can be proved in one manuscript:

$$\text{free massless scalar} \quad + \quad \text{Wick-square source} \quad + \quad \text{Wick-square readout.}$$

Its first source derivative is already quantum, singular, connected, and normalization dependent at coincidence. Its full allowed ambiguity can be classified and annihilated without deleting its nonlocal signal.

This restriction is made before any calculation. It is not a reinterpretation of VII-D's coefficient.

The role of Cheung's observation and descent results

VII-E provides an observation framework and, in the displayed version, a conditional section entitled ``A

dictionary for distributional observations" [11]. The supplied descriptions that both deny and attribute that dictionary cannot be used simultaneously. We use only its limited interpretation: observations may be blind to declared differences, but blindness does not construct the response.

Reconstruction descent is a separate condition, stated in Em01-A and XIV-J. The finite separation argument also has a classical prototype in Landmark VI. All mathematical criteria needed here are proved below.

VII-G's finite instruments distinguish selective maps, outcome effects, and implementation comparison [12]. That distinction is retained: constructing a scalar response distribution does not by itself construct an instrument measuring it.

Physical matching and complete response ambiguity

All spaces in this section are real locally convex spaces. Complex responses can be treated by adjoining their real and imaginary components.

Definition (A matched response family). Fix:

1. a physical model, preparation, source/readout calibration, and physical boundary problem;

2. a perturbative order N;

3. an admitted test space $\mathcal{E}$;

4. a nonempty class $\mathfrak{P}_0$ of physically matched prescriptions;

5. a constructed response $\mathscr{R}_N^{\mathrm{P}} \in \mathcal{E}'$ for every $\mathrm{P} \in \mathfrak{P}_0$. Every response must be expressed in the same calibrated test coordinates. Its complete difference space is

$$\mathfrak{A}_N = \operatorname{span}_{\mathbb{R}} \{\mathscr{R}_N^{\mathrm{P}} - \mathscr{R}_N^{\mathrm{Q}} : \mathrm{P}, \mathrm{Q} \in \mathfrak{P}_0\}.$$

No finite-dimensionality is assumed.

For formal truncated responses, the definition is applied coefficientwise. Equivalently, collect the finitely many coefficients into a finite direct sum of distribution spaces. An identity through order N is an identity of those

coefficients, not an uncontrolled numerical estimate of the omitted orders.

Definition (Invariant tests and observable closure). Set

$$\mathfrak{A}_N^{\perp} = \{F \in \mathcal{E} : \langle a, F\rangle = 0 \text{for every} a \in \mathfrak{A}_N\}.$$

The closure used for continuous observations is

$$\overline{\mathfrak{A}_N}^{\sigma(\mathcal{E}', \mathcal{E})},$$

where $\sigma(\mathcal{E}', \mathcal{E})$ is the weak topology generated by the actual admitted test pairings.

Theorem (Exact observable descent and nonzero detection). *Fix a reference response $\mathscr{R}_*$ in the matched family.*

1. *The pairing $\langle \mathscr{R}, F\rangle$ is independent of every prescription in $\mathfrak{P}_0$ if and only if $F \in \mathfrak{A}_N^{\perp}$.*

2. *There exists $F \in \mathfrak{A}_N^{\perp}$ with $\langle \mathscr{R}_*, F\rangle \neq 0$ if and only if*

$$\mathscr{R}_* \notin \overline{\mathfrak{A}_N}^{\sigma(\mathcal{E}', \mathcal{E})}.$$

3. *Every invariant test defines a continuous linear functional on the weak Hausdorff quotient*

$$\mathcal{E}' / \overline{\mathfrak{A}_N}^{\sigma(\mathcal{E}', \mathcal{E})}.$$

Proof. For (i), prescription independence is exactly the vanishing of the pairing on every generating difference in . Linearity extends it to their span.

If a test annihilates $\mathfrak{A}_N$, weak continuity makes it annihilate its weak closure. A nonzero pairing with $\mathscr{R}_*$ therefore implies .

Conversely, a weak neighbourhood separating $\mathscr{R}_*$ from the closure is determined by finitely many tests $F_1, \dots, F_m$. Define

$$T(u) = (\langle u, F_1\rangle, \dots, \langle u, F_m\rangle).$$

Then $T(\mathscr{R}_*) \notin T(\mathfrak{U}_N)$; otherwise an ambiguity would match all those pairings and belong to the separating neighbourhood. The linear subspace $T(\mathfrak{U}_N) \subset \mathbb{R}^m$ is closed. Choose w orthogonal to it with $w^{\mathrm{T}} T(\mathscr{R}_*) \neq 0$. The test $F = \sum_i w_i F_i$ has the required properties.

The pairing is continuous in the weak topology and vanishes on the closed subspace. The universal property of the quotient proves (iii).

Remark (Which topology is justified). An algebraically nonzero class need not be detectable by the admitted tests. Replacing the weak closure in by a stronger closure requires a separate duality theorem. No reflexivity or completeness assumption is hidden in Theorem .

Finite changes, not merely tangent directions

Example (An infinitesimal catalogue can miss a finite difference). Let $a, b \in \mathcal{E}'$ be linearly independent and consider

$$\mathscr{R}^{\theta} = \mathscr{R}^{0} + \theta a + \theta^{2} b.$$

The derivative at $\theta = 0$ detects only a, but the span of all finite differences contains both a and b. A test annihilating a alone need not give a prescription-independent value.

Thus the definition is used even when prescription changes have a nonlinear parametrization.

Propagation and reconstruction before observation

Proposition (Propagated ambiguity). *Let $P : \mathcal{U} \to \mathcal{E}'$ be a defined continuous linear map and let $u^{\mathrm{P}} \in \mathcal{U}$. If*

$$u^{\mathrm{P}} - u^{\mathrm{Q}} = c_{\mathrm{PQ}} r,$$

then

$$Pu^{\mathrm{P}} - Pu^{\mathrm{Q}} = c_{\mathrm{PQ}} Pr.$$

The observation condition is $\langle Pr, F \rangle = 0$, not $\langle r, F \rangle = 0$ unless the two objects have actually been identified.

More generally, if

$$\mathscr{R}^{\mathrm{P}} = \mathscr{R}_{*} + \sum_{v} P_{v} c_{v}^{\mathrm{P}},$$

the ambiguity is contained in the span of the images of the allowed joint changes $(c_{v}^{\mathrm{P}} - c_{v}^{\mathrm{Q}})_{v}$*. It equals a proposed image span only when those changes exhaust its generators.*

Proof. The first assertions are linearity. For the last assertion, subtract the two response formulas. Independent variation of every catalogue entry is not implied by the formula; correlations among allowed changes must be retained.

If the propagation map also changes, the exact comparison is

$$P^{\mathrm{P}} u^{\mathrm{P}} - P^{\mathrm{Q}} u^{\mathrm{Q}} = P^{\mathrm{P}}(u^{\mathrm{P}} - u^{\mathrm{Q}}) + (P^{\mathrm{P}} - P^{\mathrm{Q}}) u^{\mathrm{Q}}.$$

The second term cannot be omitted.

Finite renormalizations of an interacting theory require precisely such attention to interactions and observable identifications [4, 5]. The present proposition does not assert that the needed maps exist for the inherited characteristic graph.

Theorem (Reconstruction descent). *Let* $\mathrm{Rec} : \mathcal{P} \to \mathcal{K}$ *be a surjective linear presentation map. A linear map* $\widetilde{\mathrm{M}} : \mathcal{P} \to Y$ *factors uniquely as*

$$\widetilde{\mathrm{M}} = \mathrm{M}\mathrm{Rec}$$

if and only if

$$\ker \mathrm{Rec} \subseteq \ker \widetilde{\mathrm{M}}.$$

If $\mathcal{K}$ *has the quotient topology and* $\widetilde{\mathrm{M}}$ *is continuous, the induced map is continuous.*

Proof. Factorization annihilates every null presentation. Conversely, two presentations of the same kernel differ

by an element of ker Rec, so makes their images equal. Set $\mathrm{M}(\mathrm{Rec}p) = \widetilde{\mathrm{M}}(p)$. Surjectivity gives uniqueness, and the quotient topology gives continuity.

This is the criterion stated in Cheung's reconstruction manuscripts. It is not the free-shell equation quotient and not prescription equivalence.

Example (A null presentation with nonzero summands). For a one-dimensional coordinate q,

$$q\delta'(q) + \delta(q) = 0.$$

Indeed,

$$\langle q\delta', \psi\rangle = -\partial_q(q\psi(q))|_{q=0} = -\psi(0).$$

Tensoring with a nonzero distribution in other variables leaves the identity unchanged. A rule extracting a highest polynomial degree from the two summands need not descend to their zero reconstruction. This is the elementary mechanism used in XIV-J's null-presentation test.

Definition sheet for the quantum benchmark

Field, state, source, and readout

Set

$$M = \mathbb{R}^4, \qquad \eta = \mathrm{diag}\,(1,-1,-1,-1), \qquad z = (t,\mathbf{x}), \qquad s(z) = t^2 - |\mathbf{x}|^2, \qquad \mathrm{Box} = \partial_t^2 - \Delta_{\mathbf{x}}.$$

There is no physical reflecting or absorbing boundary.

Let φ be the real massless free scalar in its Minkowski vacuum ω_0, with

$$\omega_0(\varphi(x)\varphi(y)) = \hbar W(x-y), \qquad W(t,\mathbf{x}) = \int_{\mathbb{R}^3} \frac{e^{-i|\mathbf{p}|t + i\mathbf{p}\cdot\mathbf{x}}}{(2\pi)^3 2|\mathbf{p}|}\mathrm{d}^3\mathbf{p}.$$

Equivalently,

$$W(t,\mathbf{x}) = -\frac{1}{4\pi^2}\lim_{\varepsilon \mathrm{downarrow} 0}\frac{1}{(t-i\varepsilon)^2 - |\mathbf{x}|^2}.$$

The parameter $\hbar > 0$ is retained explicitly.

Define the calibrated vacuum-centred Wick square

$$\mathscr{Q}(x) = :\varphi(x)^2:_{\omega_0}.$$

Its field-degree coefficient is one and $\omega_0(\mathscr{Q}(x)) = 0$. For real compact tests,

$$A(f) = \int f(x)\mathscr{Q}(x)\,\mathrm{d}^4x, \qquad B(h) = \int h(y)\mathscr{Q}(y)\,\mathrm{d}^4y.$$

The external source enters with Hamiltonian density

$$\mathscr{H}_{\mathrm{src}}(y) = -\lambda h(y)\mathscr{Q}(y).$$

The initial state remains ω_0. At first source order, the retarded readout is

$$\omega_0(A^{\mathrm{ret}}_{\lambda,h}(f)) = \lambda\chi(f,h) + O(\lambda^2),$$

where, away from coincident source/readout points,

$$\chi(f,h) = \frac{\mathrm{i}}{\hbar}\int f(x)h(y)\Theta(x^0 - y^0)\omega_0([\mathscr{Q}(x), \mathscr{Q}(y)])\,\mathrm{d}^4x\,\mathrm{d}^4y.$$

The sign follows by expanding source evolution with $\mathscr{H}_{\mathrm{src}} = -\lambda h\mathscr{Q}$.

The response is connected: its two insertions are joined by vacuum contractions, and the disconnected subtraction $\omega_0(\mathscr{Q}(x))\omega_0(\mathscr{Q}(y))$ is zero. The selected perturbative order is $N = 1$ in λ. No bulk self-interaction is present.

Relative and joint test spaces

Take

$$\mathcal{E} = \mathscr{D}(M;\mathbb{R}).$$

The response kernel in relative coordinates is $\mathscr{R} \in \mathscr{D}'(M;\mathbb{R})$. Its joint kernel is the pullback

$$\mathbf{R}(x,y) = \mathscr{R}(x - y).$$

The difference map $(x,y) \mapsto x - y$ is a submersion, so this pullback exists.

For $H \in \mathscr{D}(M \times M; \mathbb{R})$, define

$$F_H(z) = \int H(y + z, y)\, \mathrm{d}^4 y.$$

This is a compact smooth test, and

$$\langle \mathbf{R}, H \rangle = \langle \mathscr{R}, F_H \rangle.$$

In particular, for $H = f \otimes h$,

$$F_{f,h}(z) = \int f(y + z) h(y)\, \mathrm{d}^4 y, \qquad \chi(f, h) = \langle \mathscr{R}, F_{f,h} \rangle.$$

Physical matching and the normalization ledger

The following data are fixed throughout the benchmark.

toprule Datum Treatment

midrule Free equation and field normalization Box $\varphi = 0$ and the two-point function ; no mass or kinetic counterterm is compared

Preparation The same Minkowski vacuum, not an arbitrary Hadamard state

Source and readout The same calibrated $\mathscr{Q}$, h, f, and source sign

Ordering coordinates Permitted only with the corresponding algebra and state transport

Coincident response normalization A source-local contact convention, classified below

Regulator Auxiliary; it must yield a limit satisfying the declared response axioms if it is used

Boundary conditions None are varied; introducing a reflecting wall changes the model

bottomrule

A multiplicative change of $\mathscr{Q}$, a source rescaling, a new vacuum, or a mass shift is not a normalization convention in this comparison.

If a different normal-ordering representation writes $\mathscr{Q}$ as a Wick square plus a smooth c-number, that c-number must be included so that the same calibrated operator and state are represented. It is not legitimate to keep the changed symbol and call the resulting preparation unchanged. The distinction between product data and a physical state is also explicit in XII-A.

The source-local contact freedom concerns the convention for coincident differentiated composite insertions. Formally, an addition

$$\frac{\hbar c}{2}\int j(y)^2 \, \mathrm{d}^4 y$$

to a connected source-functional normalization has mixed second derivative $\hbar c \int fh$. It does not change the free field equation or the zero-source composite.

If an apparatus calibration also fixes this coincident response, then c is fixed and the admissible class becomes smaller. The invariant experiment constructed below remains valid in that smaller class. No equivalence of unmatched overlapping source/readout calibrations is claimed.

The complete first-order contraction and its domain

Lemma (Connected two-insertion coefficient). *For the calibrated Wick square,*

$$\omega_0(\mathscr{Q}(x)\mathscr{Q}(y)) = 2\hbar^2 W(x-y)^2.$$

Consequently the punctured retarded response is

$$\mathscr{R}^\circ(z) = 2i\hbar\Theta(t)(W(z)^2 - W(-z)^2), \qquad z \neq 0.$$

Proof. Vacuum normal ordering removes self-contractions at each insertion. There are exactly two pairings connecting the two fields at x to the two fields at y. Each contributes $\hbar^2 W(x-y)^2$. Terms with an uncontracted normal-ordered field have zero vacuum expectation. This proves ; subtraction in the commutator gives .

Thus the first-order calculation contains the complete two-insertion vacuum sum. There is no omitted tadpole,

disconnected graph, action insertion, or scalar-sourced gravitational slot in this model.

Products of positive-frequency two-point distributions and the Wick-square construction belong to the usual microlocal Wick framework [1, 2]. The next lemma checks the subsequent retarded operation rather than assuming it.

Lemma (Punctured retarded kernel). *The distribution is defined on $M \setminus \{0\}$, has scaling degree four at the omitted point, and equals*

$$\text{boxed}\,\mathscr{R}^{\circ}(z) = \frac{\hbar}{4\pi^{3}}\Theta(t)\delta^{'}(s(z)), \qquad z \neq 0.$$

Here $\delta^{'}$ denotes differentiation with respect to the scalar argument s.

Proof. The nonzero wavefront covectors of W have one time orientation. There is no opposing pair within $W \cdot W$, so this square is a defined distributional product. The same holds for $W(-z)^{2}$.

On the nonzero future null cone, $t > 0$ and $\mathrm{d}s \neq 0$. The one-dimensional boundary-value identity

$$(u - \mathrm{i}0)^{-2} - (u + \mathrm{i}0)^{-2} = -2\mathrm{i}\pi\delta^{'}(u)$$

therefore pulls back to s. Using gives

$$W(z)^{2} - W(-z)^{2} = -\frac{\mathrm{i}}{8\pi^{3}}\delta^{'}(s(z)) \quad (t > 0).$$

Substitution proves .

Away from the origin, the commutator is supported on the nonzero null cones, where the time cut is locally smooth. At $t = 0$ and $\mathbf{x} \neq 0$ it is locally zero. Hence the time cut creates no additional punctured product problem.

Under positive dilation, s has degree two and $\delta^{'}(s)$ has degree minus four. The punctured kernel is a nonzero homogeneous distribution of degree minus four, giving scaling degree four.

toprule Operation Domain and justification

midrule W^{2} Oriented wavefront product; no opposite covectors

Retarded restriction Defined away from the origin; no multiplication at the tip is assumed

Tip extension Constructed explicitly in Section

Joint pullback The difference map is a submersion

Compact observation Proper integration in

Differentiation Weak distributional differentiation, not a sharp boundary limit

bottomrule

The wavefront assertions use the product and pullback calculus [8]. Normal-topology continuity theorems apply only on compatible cones and support-proper stages [7]; no unverified characteristic product is introduced.

An explicit extension and the full normalization class

A logarithmic reference prescription

For $\mu > 0$, let

$$L_\mu(z) = \begin{cases} \log(\mu^2 s(z)), & t > 0,\ s(z) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

This is a locally integrable function. Its definition is not a product of an undefined distribution with a characteristic function.

Define

$$\text{boxed}\,\mathscr{R}_\mu = \frac{\hbar}{64\pi^3}\text{Box}^2 L_\mu.$$

The square denotes two applications of Box, hence four derivatives in total.

Lemma (Reference extension). *The distribution is real, invariant under proper orthochronous Lorentz transformations, retarded, and extends $\mathscr{R}^\circ$. Its scaling degree is four. For every compact $K \subset M$, there is a finite constant $C_{K,\mu}$ such that*

$$\left|\left\langle \mathscr{R}_{\mu}, F\right\rangle\right| \le C_{K,\mu}\hbar\, p_{4,K}(F), \qquad \operatorname{supp} F \subset K,$$

where $p_{j,K}$ *is the maximum of the derivatives through order j on K.*

Proof. Logarithmic singularities are locally integrable transverse to the nonzero cone. At the tip, radial integration also gives a finite integral of a logarithm. Reality, Lorentz invariance, and retarded support follow from the defining function and the operator Box^2.

We verify the constant using the retarded Riesz family. In dimension four its normalization is

$$\mathrm{R}_{\alpha}(z) = \frac{2^{1-\alpha}}{\pi\Gamma(\alpha/2)\Gamma((\alpha-2)/2)}\Theta(t)\Theta(s)s^{(\alpha-4)/2}$$

on its locally integrable domain, continued holomorphically as a distribution. The identities

$$\mathrm{Box}\,\mathrm{R}_{\alpha+2} = \mathrm{R}_{\alpha}, \qquad \mathrm{R}_{0} = \delta_{0}$$

are the standard Riesz identities [6, Section 1.2].

On the punctured future cone, put $Y_{\beta}(u) = u_{+}^{\beta-1}/\Gamma(\beta)$, continued in β. Since $Y_{-1} = \delta'$ and $1/\Gamma(\alpha/2) = \alpha/2 + O(\alpha^2)$,

$$\partial_{\alpha}\mathrm{R}_{\alpha}\big|_{\alpha=0} = \frac{1}{\pi}\Theta(t)\delta'(s) \quad \text{away from} 0.$$

On the other hand,

$$\mathrm{R}_{\alpha+4} = C(\alpha+4)\Theta(t)\Theta(s)s^{\alpha/2}, \qquad C(4) = \frac{1}{8\pi}.$$

Differentiate $\mathrm{Box}^2\,\mathrm{R}_{\alpha+4} = \mathrm{R}_{\alpha}$ at zero. The derivative of C contributes only a multiple of δ_0, and the logarithmic term contributes

$$\frac{1}{16\pi}\mathrm{Box}^2\,(\Theta(t)\Theta(s)\log s).$$

It follows that restricts to $\hbar\Theta(t)\delta'(s)/(4\pi^3)$, as required.

For the estimate, weak differentiation gives

$$\langle \mathscr{R}_{\mu}, F \rangle = \frac{\hbar}{64\pi^{3}} \int_{t>0,\, s>0} \log(\mu^{2} s)\, \mathrm{Box}^{2} F(z)\, \mathrm{d}^{4} z.$$

The absolute logarithmic integral on K is finite, and $|\mathrm{Box}^{2} F| \le 16 p_{4,K}(F)$.

Finally, positive dilation of this formula gives the exact logarithmic scaling identity proved below. It implies scaling degree at most four. The nonzero punctured homogeneous restriction makes the degree exactly four.

The allowed prescriptions

Definition (Admissible first-order prescription). An admissible scalar response prescription is a real distribution $\mathscr{R}^{\mathrm{P}} = \hbar K^{\mathrm{P}}$ on M satisfying:

1. $K^{\mathrm{P}}|_{M \setminus \{0\}} = \mathscr{R}^{\circ}/\hbar$;

2. $\operatorname{supp} K^{\mathrm{P}} \subset J^{+}(0)$;

3. proper orthochronous Lorentz invariance; its associated joint kernel depends only on $x - y$;

4. $\mathrm{sd}_{0}(K^{\mathrm{P}}) \le 4$. The physical data and calibrations of Section are fixed. If a regulator is used, its limit must exist and satisfy these conditions. An unsuccessful regulator does not become a prescription merely by being named.

These are axioms for the selected scalar vacuum response at this order, not a classification of all higher time-ordered products. The allowance for a contact term is deliberate; no extra coincident-response calibration has been imposed.

Theorem (Complete first-order ambiguity). *The class in Definition is nonempty. For every fixed reference scale* $\mu_{0} > 0$*, all its members are exactly*

$$\text{boxed}\, \mathscr{R}^{\mathrm{P}} = \mathscr{R}_{\mu_{0}} + \hbar c_{\mathrm{P}} \delta_{0}, \qquad c_{\mathrm{P}} \in \mathbb{R}.$$

Every real c_{P} *is allowed. Consequently,*

$$\mathfrak{A}_{\mathrm{resp}} = \mathbb{R}\, \hbar\delta_{0}, \qquad \mathfrak{A}_{\mathrm{resp}}^{\perp} = \{F \in \mathscr{D}(M;\mathbb{R}) : F(0) = 0\}.$$

For the joint kernel, the difference is

$$\hbar(c_{\mathrm{P}} - c_{\mathrm{Q}})\delta^{(4)}(x - y).$$

Proof. Lemma supplies a member. The difference between any member and the reference is supported at the origin. A point-supported distribution is a finite sum of derivatives of δ_0. The scaling degree of $\partial^{\alpha}\delta_0$ in dimension four is $4 + |\alpha|$. The order bound therefore permits only $c\delta_0$. Reality makes c real.

Conversely, adding a real multiple of $\hbar\delta_0$ preserves every condition in Definition . Thus the catalogue is exhaustive, not merely an upper bound. Evaluation of δ_0 proves the annihilator formula, and the joint statement follows from the difference-map pullback.

This is the point-extension mechanism of microlocal renormalization [1, Theorem 5.3]. Cheung's XVI-J has the same type of one-dimensional extension freedom in its stated narrowed numerical sector. No identification of those two sectors is needed or asserted here.

Scale conversion and its exact meaning

Proposition (Scale and dilation identities). *For positive scales μ, ν,*

$$\mathscr{R}_{\mu} - \mathscr{R}_{\nu} = \frac{\hbar}{4\pi^2}\log\frac{\mu}{\nu}\,\delta_0.$$

For $F_{\ell}(z) = \ell^{-4}F(z/\ell)$, $\ell > 0$,

$$\langle \mathscr{R}_{\mu}, F_{\ell} \rangle = \ell^{-4}\Big[\langle \mathscr{R}_{\mu}, F \rangle + \frac{\hbar}{4\pi^2}\log \ell\, F(0)\Big].$$

Scale conversion is additive and every test in is insensitive to it.

Proof. The Riesz identity at $\alpha = 4$ gives

$$\mathrm{Box}^2\,(\Theta(t)\Theta(s)) = 8\pi\delta_0.$$

Subtract the logarithms in to obtain . For , change variables $z = \ell w$ in the weak integral. Four differentiations give the factor ℓ^{-4}, while $\log(\mu^2 s(\ell w)) = \log(\mu^2 s(w)) + 2\log\ell$. The same delta identity completes the calculation.

The logarithm in is a normalization conversion for this response. It is not a physical beta function or a

gravitational Wilson coefficient.

An exactly evaluated nonzero invariant

Choose $t_0 > a > 0$, and let

$$\rho(u) = \begin{cases} e^{-1/(1-u^2)}, & |u| < 1, \\ 0, & |u| \geq 1. \end{cases}$$

Set

$$b(t) = \frac{\rho((t - t_0)/a)}{\sqrt{a \int_{-1}^{1} \rho(u)^2 \, \mathrm{d}u}}.$$

Then

$$\int b(t)^2 \, \mathrm{d}t = 1, \qquad \int t \, b(t)^2 \, \mathrm{d}t = t_0.$$

The second equality uses the evenness of ρ.

Define the real compact test

$$\operatorname{boxed} F_*(t, \mathbf{x}) = (t^2 - |\mathbf{x}|^2) b(t) b(|\mathbf{x}|).$$

It is smooth because its spatial support avoids $\mathbf{x} = 0$. Its support avoids $z = 0$, although it crosses the nonzero future null cone.

Theorem (Explicit invariant quantum response). *For every admissible prescription in Definition ,*

$$\operatorname{boxed} \left\langle \mathscr{R}^{\mathrm{P}}, F_* \right\rangle = -\frac{\hbar t_0}{2\pi^2} \neq 0.$$

This value is unchanged by every finite normalization change in the complete class, not only by a selected regulator conversion.

Proof. Since $F_*(0) = 0$, Theorem removes every allowed difference. The common value can be evaluated with the punctured cone kernel.

For fixed $t > 0$, integrate spatially in polar coordinates and put $u = t^2 - r^2$. Near $u = 0$,

$$4\pi r^2 \, \mathrm{d}r = 2\pi r \, |\mathrm{d}u|.$$

The δ' pairing therefore gives

$$\begin{aligned} \int_{\mathbb{R}^3} \delta'(t^2 - r^2)(t^2 - r^2) b(t) b(r) \, \mathrm{d}^3\mathbf{x} &= -\frac{\mathrm{d}}{\mathrm{d}u}\left(2\pi\sqrt{t^2 - u}\, u\, b(t) b(\sqrt{t^2 - u})\right)\Big|_{u=0} \\ &= -2\pi t\, b(t)^2. \end{aligned}$$

Multiply by $\hbar/(4\pi^3)$, integrate in t, and apply .

Corollary (A nonzero detectable quotient class). *The class of* $\mathscr{R}_{\mu_0}$ *in*

$$\mathscr{D}'(M;\mathbb{R})/\mathbb{R}\,\hbar\delta_0$$

is nonzero and is detected by the continuous test F_*.

Proof. A response in the ambiguity subspace would pair to zero with F_*, contrary to .

The result is quantum: it is the connected response of a quantum composite in a declared vacuum and has coefficient $\hbar$. It is not an auxiliary classical metric variation.

A compact joint test with the same exact value

Choose any $v \in \mathscr{D}(M;\mathbb{R})$ with $\int v = 1$, and set

$$H_*(x,y) = v(y) F_*(x - y).$$

This is compact and smooth; gives $F_{H_*} = F_*$. Hence

$$\left\langle \mathbf{R}^{\mathrm{P}}, H_* \right\rangle = -\frac{\hbar t_0}{2\pi^2}.$$

This is an exact joint-test certificate. A general joint weight is not automatically one product $f \otimes h$. The next subsection provides a single product experiment instead.

A delayed source/readout realization

The invariant can be observed without coincident source/readout support and without treating an arbitrary joint test as a single experiment.

A useful cone estimate

Lemma (Annular cone pairing). *Let G be a smooth compact test supported at positive times away from the origin. On the cone,*

$$\langle \mathscr{R}^{\circ}, G\rangle = \frac{\hbar}{16\pi^{3}} \int_{0}^{\infty} \int_{S^{2}} \left[\frac{G(t, t\mathbf{n})}{t} + \partial_{r} G(t, t\mathbf{n}) \right] \mathrm{d}\mathbf{n}\, \mathrm{d}t.$$

If its cone sampling lies in $t \in [d, D]$, $d > 0$, *then*

$$|\langle \mathscr{R}^{\circ}, G\rangle| \le \frac{\hbar (D-d)}{4\pi^{2}} \left[d^{-1} \|G\|_{\infty} + \|\partial_{r} G\|_{\infty} \right].$$

Proof. For each $t, \mathbf{n}$, the substitution $u = t^{2} - r^{2}$ changes $r^{2}\,\mathrm{d}r$ into $r\,|\mathrm{d}u|/2$. The negative derivative at zero of $rG(t, r\mathbf{n})/2$ is

$$\frac{G(t, t\mathbf{n})}{4t} + \frac{\partial_{r} G(t, t\mathbf{n})}{4}.$$

This proves . Use area $(S^{2}) = 4\pi$ to obtain the bound.

Explicit product tests

Put $q = t_{0} - a > 0$ and

$$p_{2}(F_{*}) = \max_{|\alpha| \le 2} \sup_{z} |\partial^{\alpha} F_{*}(z)|.$$

Choose a positive number δ satisfying

$$0 < \delta < \min \left\{ a, \frac{q}{4}, \frac{t_{0}}{32 a\, p_{2}(F_{*})\, (2/q + \sqrt{3})} \right\}.$$

Let

$$\eta(u) = \frac{\rho(u)}{\int_{-1}^{1} \rho(v)\,\mathrm{d}v}, \qquad y_{0} = (2\delta, 0, 0, 0),$$

and define

$$h_{\delta}(y) = \delta^{-4} \prod_{\alpha=0}^{3} \eta\!\left(\frac{y^{\alpha} - y_{0}^{\alpha}}{\delta}\right), \qquad f_{\delta}(x) = F_{*}(x - y_{0}).$$

The source test is nonnegative and has integral one. The source lies in $\delta < y^{0} < 3\delta$, while the readout lies at $x^{0} > 2\delta + q > 3\delta$. They are strictly time ordered and disjoint.

Theorem (One delayed experiment detects the invariant). *For every admissible prescription,*

$$\chi^{\mathrm{P}}(f_{\delta}, h_{\delta}) = \left\langle \mathscr{R}^{\mathrm{P}}, F_{f_{\delta}, h_{\delta}} \right\rangle$$

has the same value and satisfies

$$\operatorname{boxed} \chi^{\mathrm{P}}(f_{\delta}, h_{\delta}) \le -\frac{3\hbar t_{0}}{8\pi^{2}} < 0.$$

The parameter δ is an experimental source width, not an ultraviolet regulator.

Proof. Write $y = y_{0} + \xi$. Formula becomes

$$F_{f_{\delta}, h_{\delta}}(z) = \int h_{\delta}(y_{0} + \xi) F_{*}(z + \xi)\,\mathrm{d}^{4}\xi, \qquad |\xi^{\alpha}| \le \delta.$$

It vanishes at zero because source and readout supports are disjoint. Every contact difference therefore pairs to zero.

Set $G = F_{f_{\delta}, h_{\delta}} - F_{*}$. The mean-value theorem gives

$$\|G\|_{\infty} \le 4\delta p_{2}(F_{*}), \qquad \|\partial_{r} G\|_{\infty} \le 4\sqrt{3}\,\delta p_{2}(F_{*}).$$

The cone is sampled only in

$$q - \delta \le t \le t_{0} + a + \delta.$$

Its lower endpoint is at least $q/2$, and the interval length is at most $4a$. Lemma yields

$$|\langle \mathscr{R}^{\circ}, G\rangle| \le \frac{4\hbar a\delta}{\pi^2} p_2(F_*)\,(2/q + \sqrt{3}) < \frac{\hbar t_0}{8\pi^2}.$$

Combine this estimate with $\langle \mathscr{R}^{\circ}, F_* \rangle = -\hbar t_0/(2\pi^2)$.

The operational interpretation is the first derivative of the vacuum expectation of the smeared Wick-square readout under a compact quadratic source pulse. The readout smearing is real and signed, as is permissible for a linear observable.

This interpretation does not construct a bounded finite-dimensional instrument for the unbounded Wick-square observable. Such an implementation would require its own measurement specification, as emphasized by the selective-map comparison in VII-G [12].

Finite banks, calibration, and error control

A complete finite-bank criterion

Let $\mathfrak{A} \subset \mathcal{E}'$ be any linear ambiguity space, possibly infinite-dimensional. Fix tests $F_1, \dots, F_m$, and define

$$T(u) = (\langle u, F_i \rangle)_{i=1}^{m}, \qquad s = T(\mathscr{R}_*).$$

The finite-dimensional evaluated ambiguity is $T(\mathfrak{A})$.

Theorem (Finite-bank separation with coverage). *Choose $a_1, \dots, a_k \in \mathfrak{A}$ such that their evaluated vectors span $T(\mathfrak{A})$, and set*

$$M_{ij} = \langle a_j, F_i \rangle.$$

There is a weight vector $w \in \mathbb{R}^m$ with

$$M^{\mathrm{T}} w = 0, \qquad w^{\mathrm{T}} s \neq 0$$

if and only if $s \notin \operatorname{col}(M)$. Every such w gives a test

$$F_w = \sum_i w_i F_i$$

annihilating the entire ambiguity $\mathfrak{A}$.

If the evaluated coverage $\operatorname{col}(M) = T(\mathfrak{A})$ *has not been proved, the same matrix calculation certifies blindness only to the listed generators.*

Proof. The orthogonal complement of $\operatorname{col}(M)$ is $\ker M^{\mathrm{T}}$. A vector in this complement has nonzero pairing with s exactly when s is not in $\operatorname{col}(M)$. For any $a \in \mathfrak{A}$, coverage gives $T(a) \in \operatorname{col}(M)$, so

$$\langle a, F_w \rangle = w^{\mathrm{T}} T(a) = 0.$$

Without coverage, this reasoning applies only to the known span.

This is the finite-certificate logic used in Cheung's earlier linear quotient analysis. It neither constructs an absent response nor proves coverage of an infinite catalogue.

Example (An exact two-test certificate for the benchmark). Choose $p \in \mathscr{D}(M;\mathbb{R})$ with $p(0) = 1$, and set

$$F_1 = F_* + p, \qquad F_2 = p.$$

Using the complete generator $a_1 = \hbar\delta_0$,

$$M = \hbar\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad w = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

The exact contrast is

$$w^{\mathrm{T}} s = -\frac{\hbar t_0}{2\pi^2}.$$

This is a certificate for the complete benchmark ambiguity because Theorem already proved its coverage.

Calibration is not automatic annihilation

Suppose finite parameters $\theta \in \mathbb{R}^k$ produce response changes $L\theta$, while calibration data change by $C\theta$. After exact calibration, the residual allowed parameters are $\ker C$.

Proposition (Residual calibrated ambiguity). *The response ambiguity remaining after this linear calibration is*

$$L(\ker C).$$

An invariant test need annihilate that residual space, not physical parameter directions that have already been fixed.

Proof. Matched parameter changes have $C\theta = 0$, and their response images are precisely $L\theta$.

For nonlinear finite matching, replace this formula by the actual finite differences satisfying the calibration conditions. Interaction and observable matching under finite renormalization must be performed together [4].

Minimum-variance invariant weights

Proposition (Optimal weights for a covered bank). *Assume the complete evaluated ambiguity is* $\operatorname{col}(M)$ *and that the measurement covariance* Σ *is positive definite. Set*

$$C = \Sigma^{-1/2} M, \qquad q = \Sigma^{-1/2} s,$$

and let Π *be the Euclidean orthogonal projection onto* $\ker C^{\mathrm{T}}$. *If* $\Pi q \neq 0$, *the unique weights minimizing* $w^{\mathrm{T}} \Sigma w$ *subject to*

$$M^{\mathrm{T}} w = 0, \qquad s^{\mathrm{T}} w = 1$$

are

$$w_* = \frac{\Sigma^{-1/2} \Pi q}{\|\Pi q\|^2}, \qquad w_*^{\mathrm{T}} \Sigma w_* = \frac{1}{\|\Pi q\|^2}.$$

They are unchanged when s *is replaced by* $s + M\theta$.

Proof. Put $y = \Sigma^{1/2} w$. The constraints become $C^{\mathrm{T}} y = 0$ and $q^{\mathrm{T}} y = 1$, and the variance is $\|y\|^2$. On the permitted subspace, $q^{\mathrm{T}} y = (\Pi q)^{\mathrm{T}} y$. Cauchy–Schwarz gives the minimum at $y = \Pi q / \|\Pi q\|^2$. Finally, $\Sigma^{-1/2} M\theta \in \operatorname{col}(C)$, which Π annihilates.

This is a mathematical measurement-design result. It assumes a specified covariance model and does not derive that model from the field theory.

Imperfect annihilation and numerical errors

Proposition (Uniform sensitivity bound). *Suppose an actual evaluated prescription difference is $M\theta$ with $\|\theta\| \le C_0$. Then*

$$|\Delta I_w| \le C_0 \|M^{\mathrm{T}} w\|.$$

If a numerical matrix $\widehat{M}$ satisfies $\|M - \widehat{M}\|_{\mathrm{op}} \le \varepsilon_M$ and $\widehat{M}^{\mathrm{T}} w = 0$, then

$$|\Delta I_w| \le C_0 \varepsilon_M \|w\|.$$

If each estimated response pairing has deterministic error at most ε_i, the contrast error is at most $\sum_i |w_i| \varepsilon_i$.

Proof. Use

$$\Delta I_w = w^{\mathrm{T}} M \theta = (M^{\mathrm{T}} w)^{\mathrm{T}} \theta$$

and Cauchy–Schwarz. The matrix and pairing estimates follow from the operator norm and triangle inequality.

Without a bound on the allowed coefficients, approximate annihilation does not give a uniform prescription-independence estimate.

Smoothing a constructed response

Proposition (Regulator smoothing after construction). *Let ρ_ε be a real compact smooth approximate identity with integral one, support in a Euclidean ball of radius ε, and uniformly bounded L^1-norm. For a fixed compactly supported test F whose support avoids the origin,*

$$\left\langle \mathscr{R}^{\mathrm{P}} * \rho_\varepsilon, F \right\rangle \to \langle \mathscr{R}^{\mathrm{P}}, F \rangle.$$

For sufficiently small ε, the contact contribution vanishes exactly, uniformly in c_{P}. On a fixed support stage the remaining error is bounded by

$$C \hbar \varepsilon \, p_5(F),$$

where C depends on a reference prescription and the uniform mollifier bound, not on c_{p}.

Proof. Pair convolution with the reflected test convolution. The mean-value theorem bounds its difference from F, through four derivatives, by a constant times $\varepsilon p_5(F)$. Apply on a common enlarged compact.

The difference between any prescription and the reference is $\hbar c_{\mathrm{p}} \delta_0$. After convolution it is supported in the radius-ε ball, which is disjoint from supp F for sufficiently small ε. Its pairing is therefore exactly zero.

This proposition smooths an already defined distribution. It is not a construction of an otherwise undefined graph product, and finite-ε smoothing need not preserve exact causal support.

The benchmark is an exact first derivative. No nonperturbative bound on the $O(\lambda^2)$ term in is claimed. Such a bound, calibration uncertainty, and apparatus noise are separate from and .

Why the same shortcuts fail in a more general response

The benchmark has only one contact direction because its locality, fixed physical data, punctured kernel, and scaling-degree condition have been proved sufficient. That conclusion must not be transferred to another graph.

Finite normal order need not mean finite dimension

Proposition (An infinite ambiguity at fixed normal order). *On* $\mathbb{R}_y^d \times \mathbb{R}_n^k$*, the distributions*

$$a(y)\delta(n), \qquad a \in \mathscr{D}(\mathbb{R}^d),$$

have normal order zero along $\{n = 0\}$*, yet form an infinite-dimensional vector space. Even if their tangential support is restricted to one point, allowing arbitrary tangential distribution order remains infinite-dimensional.*

Proof. Independent smooth functions a give independent distributions, as follows by pairing with a test equal to one near $n = 0$. For the point-supported claim, the distributions

$$\partial_y^\alpha \delta(y - y_0)\delta(n), \qquad \alpha \in \mathbb{N}^d,$$

are independent, as monomial tests with a cutoff isolate their coefficients.

This is the distinction emphasized in Cheung's punctured normal-jet analysis. It is compatible with the finite-dimensional point-extension result used in the benchmark: the latter also bounds the total scaling degree.

A local action correction can have a nonlocal image

Example (Separated source and readout do not kill every contact). Let E^{R} be the wave retarded operator and c a smooth compact function. A first-order change of the wave operator by multiplication with c changes its inverse by

$$\delta E^{\mathrm{R}} = -E^{\mathrm{R}} c E^{\mathrm{R}}.$$

For compact tests f, h, the measured change is

$$-\int c(y)(E^{\mathrm{A}} f)(y)(E^{\mathrm{R}} h)(y)\, \mathrm{d}^4 y.$$

The pairing is defined on smooth propagated tests and may be nonzero with separated f, h. Choose h and f so their retarded and advanced responses are nonzero at one intermediate point; continuity permits a compact c of fixed sign in a sufficiently small neighbourhood.

Thus external separation eliminates the benchmark's completely classified coincidence ambiguity, not an arbitrary action counterterm. Cheung's Em02-E makes the same local-to-nonlocal distinction for its spatial Green-kernel model.

Late source agreement does not erase preparation memory

Example (An elementary zero-data memory term). Choose $F \in C_c^\infty((0, \varepsilon))$, $F \geq 0$, $F \equiv 0$. The sources F and zero agree for $t \geq \varepsilon$, but their zero-data response difference is

$$z(t) = \int_0^t (t - s) F(s)\, \mathrm{d}s.$$

For $t \geq \varepsilon$,

$$z(t) = t \int F - \int sF > 0.$$

This is the memory test recorded in XVI-B. Source agreement, response agreement, and observable agreement are separate assertions.

A finite bank is not an injective reconstruction

Example (An invisible nonzero distribution). Given finitely many compact tests $F_1, \dots, F_m$ on an open set, choose a nonzero smooth compact function u orthogonal to their finite-dimensional span. Then

$$\langle u, F_i \rangle = 0 \quad \text{for every} i, \qquad u \neq 0.$$

A vanishing bank therefore does not imply a zero response distribution. Theorem tests a declared quotient on that bank; it is not a general inverse theorem.

The VII-D conversion and the inherited analytic boundary

What can be transported from a numerical conversion

Suppose the supplied VII-D family satisfies

$$u_\mu^a - u_\mu^b = c_{ab} r$$

as distributions on its joint test space. This is a comparison input about that numerical construction [10], not the benchmark kernel of this paper.

Proposition (Conversion consistency and derivative transfer). *If $r \neq 0$, the exact relation implies*

$$c_{ab} + c_{bc} = c_{ac}.$$

If a fixed linear response map P has been constructed, its conversion is $c_{ab} P(r)$.

Let T be a differential operator of order d with fixed smooth coefficients on a compact test stage. An estimate

$$|\langle \nu_R, F \rangle| \leq C R^{-1} p_{K,64}(F)$$

implies

$$\left|\left\langle T^{\mathrm{T}} v_R, F \right\rangle\right| \le C^{'} R^{-1} p_{K,64+d}(F).$$

Proof. Subtract the three distributions to obtain $(c_{ab} + c_{bc} - c_{ac})r = 0$, and use $r \neq 0$. The response statement is Proposition . For the last estimate, pair v_R with TF and use the compact-stage bound $p_{K,64}(TF) \le C_T p_{K,64+d}(F)$.

This explains the test-order loss under the numerical transfer discussed in IX-B. It does not factor the numerical finite part through a missing sharp binary multiplication.

If the response includes independent action, composite, preparation, or boundary-normalization changes, their actual response images must be added. Relation alone does not prove that the complete ambiguity is $\mathbb{R}P(r)$.

The sharp-domain obstruction is not removed by notation

The supplied IX-A statement gives a continuous coordinate separating the unchanged regular approximants from the proposed sharp insertion. The supplied IX-B statement distinguishes a ternary numerical finite part from a second sharp binary product. The VII-C obstruction concerns its stated extension and continuity hypotheses [9].

None of those problems is used in the benchmark. Conversely, the benchmark's explicit distribution does not construct the missing sharp insertion, change its admission topology, or contradict those results.

The gravitational branch remains separate

The supplied XII-A–F chain has the following scope:

1. XII-A constructs a free-shell carrier and ordering transport.

2. XII-B distinguishes density weighting from source weighting.

3. XII-C selects the cubic-only density modification and proves its actual Euler source.

4. XII-D constructs a smooth classical retarded response and an admitted completed insertion.

5. XII-E assembles the second metric slot with the sign $\Phi_{\mathrm{D}}^{(2)} = \mathscr{A}_2^{\mathrm{sc}} - \mathscr{C}_{3,\mathrm{D}}$.

6. XII-F evaluates a nonzero auxiliary remainder even when the propagated companion vanishes. These are distinct claims.

In particular, the supplied XII-F witness has the signed value

$$-8\int f(t,\mathbf{x})\int_0^t (t-s)^2 a(s)\,\mathrm{d}s\;\mathrm{d}^4x < 0.$$

It is a classical variation of a named off-shell lift tested on an initial free-shell configuration. It is not the response , not a normalization difference, and not a quantum Ward identity.

The affine second-slot identity already determines the minus sign. An admission theorem alone does not prove a kernel nonzero, and a second co-transformation row must not be added to a slot whose variation already contains that transformation.

The chronological characteristic-map stop

The supplied XIV-H statement leaves an internal amplitude obligation open. XIV-I subsequently supplies its named internal amplitude construction at I2, but leaves the full joint characteristic map open at I3. XIV-J supplies reconstruction and an off-characteristic reference for the supported sector, but leaves the full map open at J2.

Thus it would be inaccurate to treat the earlier H2 statement as if XIV-I had made no progress. It would also be inaccurate to replace the later missing joint map by the already defined internal tree kernels.

The remaining obligations for that inherited target are:

1. the complete joint characteristic map, with reconstruction descent;

2. a joint section and controlled complete remainder;

3. the required transverse estimates;

4. contact compatibility;

5. the output wavefront theorem;

6. admission of the actual quantum coefficient. No step is supplied by the scalar benchmark. Abstract extension or descent interfaces are not that graph operation.

par **Two different final statuses** par **The local benchmark of this paper: Constructed.** Its full first-order

ambiguity and a nonzero invariant quantum pairing are proved.

The inherited reflected-triangle/gravitational target: Unresolved on the supplied data. No embedding of VII-D's coefficient into a complete quantum response is claimed, and no full characteristic coefficient is admitted by this paper. par

Final theorem and interpretation

Theorem (VII-H release). *For the free massless scalar, vacuum, calibrated Wick square, source sign, test space, and prescription class declared in Sections –:*

1. *the complete first source-order connected vacuum response exists as a real retarded distribution;*

2. *every permitted finite normalization difference is a real multiple of* $\hbar\delta_0$*;*

3. *the test* F_* *of annihilates all such differences and has the exact nonzero value* $-\hbar t_0/(2\pi^2)$*;*

4. *the delayed product experiment is prescription independent and strictly nonzero;*

5. *the finite-bank, calibration, optimization, and error statements apply with the complete ambiguity already classified. This is a normalization-independent connected quantum response within the declared model and order. It is not a completion of the inherited gravitational quantum-response programme.*

Proof. Lemmas , , and construct the response. Theorem gives the complete finite comparison. Theorems and give the invariant and its delayed realization. Section proves the remaining assertions.

For this benchmark the programme is therefore realized as

$$\text{boxed}\quad \begin{aligned} \mathscr{R}^{\circ} &= \frac{\hbar}{4\pi^3}\Theta(t)\delta'(t^2-|\mathbf{x}|^2),\\ \mathscr{R}_{\mu} &= \frac{\hbar}{64\pi^3}\mathrm{Box}^2[1_{\{t>0,\ s>0\}}\log(\mu^2 s)],\\ \mathscr{R}^{\mathrm{P}} - \mathscr{R}^{\mathrm{Q}} &= \hbar(c_{\mathrm{P}}-c_{\mathrm{Q}})\delta_0,\\ F_*(0) = 0, &\qquad \left\langle \mathscr{R}^{\mathrm{P}}, F_* \right\rangle = -\frac{\hbar t_0}{2\pi^2} \neq 0. \end{aligned}$$

The surviving signal is not obtained by choosing a preferred normalization. It is obtained by classifying the entire permitted difference space and testing outside it.

For a later interacting or gravitational model, the same quotient logic becomes available only after its response and its complete physically matched ambiguity have been constructed. Neither a finite graph coefficient, a selected extension, a classical shell identity, nor a finite readout model substitutes for those tasks.

Declarations

Model selection. The affirmative result concerns a separately declared local free-field benchmark. It is not attributed to VII-D or to the XII gravitational carrier.

Quantum scope. The constructed quantity is a connected scalar vacuum susceptibility at first order in an external source. No higher source-order renormalization catalogue, interacting gravitational differential, or gravitational Ward identity is asserted.

Matching scope. The free equation, state, field normalization, source, and zero-source composite calibration are fixed. The remaining contact convention is explicitly declared. Changes of physical state, mass, boundary conditions, or unmatched detector normalization are excluded.

Operational scope. The delayed experiment specifies a source pulse and smeared readout whose linear response is nonzero. No finite measurement instrument for the Wick-square observable is constructed.

Dependency scope. The primary literature supplies the Wick and distributional framework and the stated Riesz identities. The response coefficient, normalization classification, invariant evaluation, delayed experiment, and linear separation arguments are proved here. Cheung's manuscripts are cited with their stated domains; their unresolved constructions are not treated as theorems of existence.

Verification. No successful pdflatex execution, proof-assistant verification, numerical execution, independent referee review, or journal acceptance is asserted.

Book expansion notes (t433u)

The normalization-independent connected response benchmark is not an interacting product and not a quantum

coefficient. I-B-G-Q and I-B-G-Q-B remain separations. Observation-boundary and Enter calculus records: [11], [18].

Standing ledger. Not spacetime emergence; not Einstein dynamics; not a Page curve, island, bounce, vacuum-energy selection, or de Sitter state as an Enter theorem; not a derived metric. O.1 stays NOT_CLOSED. XVI-N is not discharged. XV-B is not opened. No quantum coefficient is admitted. Host C is not created. Dom (T) remains undetermined; J2 remains unresolved. Problems 6, 7, and 9 are named, not solved. Gate 0 and ST4b stay NOT_SUPPLIED. Host-A stays NAMED_COMPARISON_ONLY $\{GI, CB\}$. The shredder is not Gibbons–Hawking. Cited sheets are source records, not premises. Conjectures stay conjectures. wired=false.

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