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Landmark VII-I Finite-time harmonic-coordinate insertions: three repair routes, finite-width free products, and sharp-prescription incompatibility

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Abstract

We examine the passage from a finite-time relational scalar to a quantum insertion, compatible products, and an interacting physical observable. The fixed classical prescription uses coordinate-time Cauchy derivatives; replacing them by derivatives along a metric-dependent unit normal would change the observable.

All three proposed repair routes are developed. Field-argument convolution gives a regular free algebra at positive width, but an explicit positive first-order defect excludes covariance under the unchanged plateau gauge action. A smooth divergence-form density preparation gives compact microcausal polynomial insertions through second coordinate order. A tree-composition argument supplies their free products, adjoints, and ordering-stable generated sector. Its sharp limit, however, has a calculated nonzero transmission velocity and is not the original zero-Cauchy prescription.

For the sharp route we construct a minimal boundary-sensitive classical carrier and prove that unchanged regular approximants do not converge in its topology. Separately, the VII-C/VII-D numerical asymptotic gives a quantitative incompatibility between old-product agreement, convergent sharp admission, bounded-set hypocontinuity, and continuous extraction of the triangle coefficient. A finite part cannot satisfy all those requirements.

Auxiliary coordinates are classically equivalent to the prescribed Cauchy construction when both equations and data are imposed. Quantum elimination requires inherited cross-contractions; an independent central-coordinate prescription fails that test. Linear sharp coordinates do have exact smooth representatives on the free shell, yielding a genuine linear free quantum sector.

Presentation descent, associativity, and relative classical gauge closure are proved at their stated levels. The sharp nonlinear interacting quantum observable, its quantum differential, and its complete normalization catalogue remain unresolved. No missing characteristic composition is supplied by a classical carrier, an internal tree kernel, or a numerical subtraction.

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par **Release and proof boundary** par **Constructed:** two explicit smoothing comparisons; a finite-width microcausal insertion and free polynomial algebra; a minimal boundary-sensitive classical carrier; classical auxiliary equivalence; and a linear free-shell quantum repair.

Proved incompatible: the unchanged off-shell sharp insertion with the microcausal polynomial domain; the

convolution comparison with the unchanged classical gauge action; and the specified continuous sharp Wick-extension requirements.

Unresolved: an associative renormalized sharp nonlinear sector outside those requirements, mixed time-ordered products, an interacting quantum gauge differential, and a physical correlation of the original sharp relational observable.

All positive constructions are labelled by the observable and domain they actually use. par

The target and the requirement ledger

The desired chain is

$$\text{boxed classical functional} \rightarrow \text{quantum insertion} \rightarrow \text{compatible products} \rightarrow \text{interacting gauge class.}$$

These implications require different theorems.

We retain the restricted gauge domain of Cheung's VII-A [1]. The boundary non-admission in VII-B, the normal-topology obstruction in VII-C, and the numerical finite part in VII-D concern different requirements [2, 3, 4]. Their coexistence is not a contradiction.

Relational-coordinate calculations and causal propagator constructions provide comparison methods [18, 19]. Equicausal functionals provide uniform control for their admitted functional class [12]; they do not admit a kernel already violating the underlying microcausal exclusion. Off-shell deformation quantization, causal perturbation theory, and renormalized BV theory provide the surrounding frameworks [13, 14, 15, 16, 17], not an automatic construction for the present sharp insertion.

Four levels

Definition (Levels used in this paper).

1. An *insertion* is an admitted algebra element with a specified classical symbol.

2. A *free sector* has defined products, associativity, adjoint, closure, and stated ordering comparison.

3. An *interacting physical sector* additionally has the required mixed time-ordered products and a nilpotent quantum gauge differential whose cohomology contains the insertion.

4. A *state-level prediction* requires a specified state or representation and a controlled evaluation in that sector.

A coefficient distribution is not an insertion, and an insertion is not automatically an interacting cohomology class.

toprule Requirement Treatment

midrule Original sharp Cauchy prescription Fixed explicitly; not replaced silently

Off-shell classical equality Retained in the obstruction; modified only in named repairs

Old free Wick products Retained at finite width and in the incompatibility theorem

Convergence of unchanged sharp approximants Excluded in the boundary-sensitive topology; incompatible with the specified Wick-extension assumptions

Quantum presentation independence Required for sharp products; not inferred from a classical identity

Plateau gauge domain Retained; no unrestricted diffeomorphism claim

Quantum Ward identities and interactions Not supplied by free admission

State positivity Not asserted for the indefinite graviton covariance

bottomrule

One classical observable and its actual Cauchy data

Conventions

Throughout our constructions,

$$M = \mathbb{R}^4, \qquad \eta = \mathrm{diag}\,(1,-1,-1,-1), \qquad \mathrm{Box} = \partial_t^2 - \Delta_{\mathbf{x}}, \qquad \Sigma = \{t = 0\}.$$

Put $M_+^\circ = \{t > 0\}$. The retarded and advanced inverses of Box are $E_0^{\mathrm{R}}, E_0^{\mathrm{A}}$, and $E_0 = E_0^{\mathrm{R}} - E_0^{\mathrm{A}}$.

The zero-data solution operator is

$$\mathrm{Box}\,\mathrm{E}_0 j = j \quad (t > 0), \qquad \mathrm{E}_0 j|_\Sigma = \partial_t \mathrm{E}_0 j|_\Sigma = 0.$$

On future smooth sources it is

$$\mathrm{E}_0 j = E_0^{\mathrm{R}}(\theta j)|_{\{t \geq 0\}}.$$

For compact observation sets, no spatial decay is required: finite propagation reduces each calculation to a compact causal region [8].

Let

$$q = \eta + \kappa h + \kappa^2 k \bmod \kappa^3.$$

The coefficient k is metric-jet bookkeeping, not a second independent quantum metric field.

Set

$$\tau(h) = \eta^{\mu\nu} h_{\mu\nu},$$

$$a_1^{\mu\nu}[h] = \tfrac{1}{2}\tau(h)\eta^{\mu\nu} - h^{\mu\nu},$$

$$a_2^{\mu\nu}[h] = h^\mu{}_\rho h^{\rho\nu} - \tfrac{1}{2}\tau(h)h^{\mu\nu} + \Big(\tfrac{1}{8}\tau(h)^2 - \tfrac{1}{4}h_{\rho\sigma}h^{\rho\sigma}\Big)\eta^{\mu\nu},$$

$$C^\mu h = \partial_\nu h^{\nu\mu} - \tfrac{1}{2}\partial^\mu \tau(h) = -\partial_\nu a_1^{\nu\mu}[h].$$

The coordinates solve

$$\partial_\alpha\Big(\sqrt{-q}\, q^{\alpha\beta}\partial_\beta \mathrm{X}_q^\mu\Big) = 0, \qquad \mathrm{X}_q^\mu|_\Sigma = x^\mu, \qquad \partial_t \mathrm{X}_q^\mu|_\Sigma = \delta_0^\mu.$$

Their formal expansion is

$$\mathrm{X}_q^\mu = x^\mu + \kappa w^\mu[h] + \kappa^2 v^\mu[h,k],$$

$$w^\mu[h] = \mathrm{E}_0 C^\mu h,$$

$$v^\mu[h,k] = \mathrm{E}_0 C^\mu k - \mathrm{E}_0\Big[\partial_\alpha a_2^{\alpha\mu}[h] + \partial_\alpha\Big(a_1^{\alpha\beta}[h]\partial_\beta w^\mu[h]\Big)\Big].$$

Metric-dependent normal data would be different

Proposition (Normal-data correction). *The data in are not equivalent to fixing* $n_q^\alpha \partial_\alpha \mathrm{X}_q^\mu = \delta_0^\mu$ *on* Σ*, where* n_q *is the future unit normal. For the latter prescription,*

$$\partial_t w^\mu|_\Sigma = -n_1^\mu, \qquad n_1^0 = -\tfrac{1}{2}h^{00}, \qquad n_1^i = -h^{i0}.$$

Proof. For the surface $t = 0$,

$$n_q^\mu = \frac{q^{\mu 0}}{\sqrt{q^{00}}} = \delta_0^\mu + \kappa n_1^\mu + O(\kappa^2).$$

Expanding the fixed normal derivative, and using $\partial_\alpha x^\mu = \delta_\alpha^\mu$, gives $\partial_t w^\mu + n_1^\mu = 0$. The inverse-metric expansion gives the displayed n_1.

Thus no argument below substitutes normal-derivative data for the coordinate-time prescription inherited from VII-A.

The smeared relational functional

For real $f \in C_c^\infty(M_+^\circ)$, define formally

$$\mathscr{O}_f[q,\varphi] = \int f(y)\varphi(\mathrm{X}_q^{-1}(y))\,\mathrm{d}^4y.$$

The leading map is the identity, so formal inversion is unique. Its coefficients are

$$\mathscr{O}_f^{(0)} = \int f\varphi,$$

$$\mathscr{O}_f^{(1)} = \mathscr{B}_f = -\int f w^\mu[h]\partial_\mu\varphi,$$

$$\mathscr{O}_f^{(2)} = \int f\Big[\big(w^\nu\partial_\nu w^\mu - v^\mu\big)\partial_\mu\varphi + \tfrac{1}{2}w^\mu w^\nu\partial_\mu\partial_\nu\varphi\Big].$$

All integrals in this paper use d^4x unless indicated otherwise.

For $K = \operatorname{supp} f$, the metric dependence lies in

$$D_f = J^-(K) \cap \{t \geq 0\},$$

a compact set. Nested retarded operations do not enlarge this overall causal past.

Proposition (Classical presentation identity). *Through order* κ^2,

$$\mathcal{O}_f[q,\varphi] = \int \det\left(D\mathrm{X}_q(x)\right) f(\mathrm{X}_q(x))\varphi(x)\,\mathrm{d}^4x.$$

The determinant has leading term 1*; no absolute-value ambiguity arises in the formal orientation-preserving expansion.*

Proof. Formal change of variables gives the identity. Explicitly,

$$\det D\mathrm{X}_q = 1 + \kappa\partial_\mu w^\mu$$

$$+\kappa^2\Big[\partial_\mu v^\mu + \tfrac{1}{2}\Big((\partial_\mu w^\mu)^2 - \partial_\mu w^\nu \partial_\nu w^\mu\Big)\Big].$$

Multiply this by the Taylor expansion of $f(x + \kappa w + \kappa^2 v)$. At order one, integration by parts gives $-\int f w^\mu \partial_\mu \varphi$. At order two, two integrations by parts give precisely .

The observation factor has compact support strictly above Σ. These integrations occur in the observation variable. They do not delete the initial-surface terms obtained when a coordinate kernel is subsequently transposed in a metric variable.

Restricted gauge transformations

Fix a scalar-action switch g, a nonempty plateau U, and

$$g \in C_c^\infty(M_+^\circ), \qquad \overline{U} \subset \mathrm{int}\,\{g = 1\}, \qquad \mathfrak{g}_U = C_c^\infty(U; TM).$$

The group is generated by flows of these vector fields. Its second-order formal pullbacks are

$$F_\kappa^* = \exp\left(\kappa\mathscr{L}_\xi + \kappa^2\mathscr{L}_\zeta\right) \bmod \kappa^3, \qquad \xi, \zeta \in \mathfrak{g}_U.$$

They are the identity near Σ.

For the unswitched coordinate equation,

$$\mathrm{X}_{F^{*}q} = \mathrm{X}_{q} \circ F$$

by naturality and Cauchy uniqueness. Therefore

$$\mathscr{O}_{f}[F^{*}q, F^{*}\varphi] = \mathscr{O}_{f}[q, \varphi] \bmod \kappa^{3}.$$

For the switched scalar solution with fixed free input, the additional plateau intertwiner is the theorem of VII-A [1]. It is not unrestricted fixed-switch covariance.

At the background coordinate map, preserving fixed values and fixed coordinate-time derivatives requires

$$\xi|_{\Sigma} = 0, \qquad \partial_{t}\xi|_{\Sigma} = 0$$

at first order. Vanishing only of $\xi|_{\Sigma}$ is insufficient. Support in U supplies a stronger sufficient condition.

The sharp kernel and the off-shell admission obstruction

Let $\mathcal{P}_{\mathrm{mc,\,pol}}$ be the finite polynomial space with compact, species-symmetric kernels u_{n} satisfying

$$\mathrm{WF}\,(u_{n}) \cap \left(\overline{V}_{+}^{n} \cup \overline{V}_{-}^{n}\right) = \varnothing,$$

where each cone contains its zero covector and the total zero covector is omitted.

For a linear coordinate test $a_{\mu} \in C_{c}^{\infty}(M_{+}^{\circ})$, put

$$\ell_{a}[h] = \int a_{\mu}(\mathrm{E}_{0}Ch)^{\mu}, \qquad b = E_{0}^{\mathrm{A}}a.$$

The formal adjoint of C is

$$(C^{*}b)^{\alpha\beta} = -\partial^{(\alpha}b^{\beta)} + \tfrac{1}{2}\eta^{\alpha\beta}\partial^{\mu}b_{\mu}.$$

Proposition (Exact boundary kernel). *The compact distribution* $q_{a} = L^{*}a$, $L = \mathrm{E}_{0}C$, *is*

$$q_{a}^{\alpha\beta} = \theta(C^{*}b)^{\alpha\beta} + \delta(t)\mathrm{B}^{\alpha\beta}(b|_{\Sigma}),$$

$$\mathrm{B}^{\alpha\beta}(z) = -n^{(\alpha}z^{\beta)} + \tfrac{1}{2}\eta^{\alpha\beta}n^{\mu}z_{\mu}, \qquad n = \mathrm{d}t.$$

There is no $\delta'(t)$ *term. The map* B *is injective. If* $b(0,\mathbf{x}_0) \neq 0$, *every nonzero normal covector* $(0,\mathbf{x}_0;\tau\,\mathrm{d}t)$ *belongs to* $\mathrm{WF}(q_a)$.

Proof. Transpose $E_0^{\mathrm{R}}(\theta Ch)$ against a:

$$\ell_a[h] = \left\langle C^*(\mathrm{theta}\mathrm{E}_0^{\mathrm{A}}\, a), h \right\rangle.$$

The product rule gives . Its support is in $J^-(\operatorname{supp} a) \cap \{t \geq 0\}$.

The components

$$\mathrm{B}^{00}(z) = -\tfrac{1}{2}z_0, \qquad \mathrm{B}^{0i}(z) = \tfrac{1}{2}z_i$$

prove injectivity.

Locally, a nonzero component has the form $\delta(t)v(\mathbf{x}) + \theta(t)r(t,\mathbf{x})$, with $v(\mathbf{x}_0) \neq 0$ and smooth coefficients. After localization, the first term has normal Fourier order zero, whereas the second has order at most minus one at fixed tangential frequency. The leading nonzero term cannot cancel. Thus rapid decrease fails in both normal directions. These are actual timelike conormals, not merely an upper bound.

Theorem (Off-shell non-admission, with a universal smearing witness). *For every nonzero real* $f \in C_c^\infty(M_+^\circ)$,

$$\mathscr{B}_f \notin \mathcal{P}_{\mathrm{mc,\,pol}}.$$

No element of $\mathcal{P}_{\mathrm{mc,\,pol}}[[\hbar]]$ *has classical symbol equal to* $\mathscr{B}_f$ *on all smooth configurations.*

Proof. The metric derivative is

$$(\mathscr{B}_f)'_h = -L^*(f\,\mathrm{d}\varphi).$$

Choose

$$\varphi_f = -\partial_t(tf).$$

For $b = E_0^{\mathrm{A}}(f\,\mathrm{d}\varphi_f)$, integration of $\mathrm{Box}\, b_0 = f\partial_t\varphi_f$ over space gives

$$\int b_0(0,\mathbf{x})\,\mathrm{d}^3\mathbf{x} = \int tf\partial_t\varphi_f\,\mathrm{d}^4x = \int (\partial_t(tf))^2\,\mathrm{d}^4x > 0.$$

The advanced solution has zero final data, which fixes the first equality. The last expression is nonzero because compact support and $\partial_t(tf) = 0$ would imply $f = 0$.

Proposition gives a timelike wavefront component of the metric derivative. Partial contraction of an admitted microcausal polynomial kernel with smooth configurations cannot produce such a component: adjoining zero covectors in the contracted slots would violate the original cone exclusion. This proves non-admission.

An off-shell-equal classical symbol has the same derivative. Positive powers of $\hbar$ cannot alter that contradiction.

This reproduces the relevant VII-B mechanism with its hypotheses visible [2]. It does not prove universal non-quantizability.

toprule Kernel feature Actual consequence

midrule Compact causal support The classical polynomial exists

Boundary delta in a contracted derivative Timelike conormals exclude off-shell microcausal admission

Direct and reflected null branches in the joint kernel A larger characteristic-cone allowance requires its own test

Reflected mixed covariance The subsequent numerical triple has a logarithmic coefficient

Finite numerical subtraction Does not establish a continuous binary multiplication

bottomrule

Functional differentiability, distributional existence, contraction existence, and continuity are different entries in this atlas.

Route B1a: field-argument convolution

Choose a real nonnegative even Euclidean-radial compact mollifier ρ, with integral one. Let

$$S_\varepsilon u = \rho_\varepsilon u, \qquad \rho_\varepsilon(x) = \varepsilon^{-4}\rho(x/\varepsilon).$$

At the level of the coordinate construction, define

$$q_\varepsilon = \eta + \kappa S_\varepsilon h + \kappa^2 S_\varepsilon k, \qquad \mathscr{O}^{\mathrm{conv}}_{f,\varepsilon}[h,k,\varphi] = \mathscr{O}_f[q_\varepsilon, S_\varepsilon \varphi].$$

This is not graph-only smoothing.

Proposition (Positive-width regular sector). *Through coordinate order two, every coefficient of* $\mathscr{O}^{\mathrm{conv}}_{f,\varepsilon}$*, for fixed* $\varepsilon > 0$*, has a smooth compact kernel. Consequently these coefficients belong to* $\mathcal{P}_{\mathrm{reg,pol}}$ *and have all finite free Wick products in the ordinary polynomial algebra.*

On fixed smooth configurations the coefficients converge to their sharp classical counterparts as $\varepsilon \mathrm{downarrow}\, 0$*. This is not convergence of their quantum products.*

Proof. Each sharp coefficient is a continuous finite polynomial in smooth fields, with compact multilinear kernel by the kernel theorem and finite propagation. Convolution in every field slot makes that kernel smooth and enlarges support only by a fixed compact ε-neighbourhood.

On fixed smooth configurations, convolution converges in all smooth seminorms on the relevant causal compact. Continuous dependence of the coefficientwise Cauchy problems then proves the classical convergence.

The finite-width free construction changes the observable. It cannot be assumed covariant under the original field action.

Theorem (Explicit convolution-covariance defect). *There are admissible plateau parameters and free massless scalar data for which the first-order variation of* $\mathscr{O}^{\mathrm{conv}}_{f,\varepsilon}$*, under the unchanged pullback action on the original fields, is strictly positive.*

In a suitable compact dependence neighbourhood, one can choose

$$\xi = t\partial_t, \qquad \varphi = \tfrac{1}{2}(t^2 + (x^1)^2), \qquad h = 0,$$

with a compact realization of $\xi \in \mathfrak{g}_U$*, and obtain*

$$\Delta_\xi \mathscr{O}^{\mathrm{conv}}_{f,\varepsilon} = \varepsilon^2 m_{00} \int f \, \mathrm{d}^4x > 0, \qquad m_{00} = \int z_0^2 \rho(z) \, \mathrm{d}^4 z.$$

Here $f \geq 0$, $f \equiv 0$, *and the sampled region is above the initial surface.*

Proof. Choose the compact parameter support strictly above Σ, and ε small enough that its smearing still vanishes near Σ. Constant-coefficient differentiation commutes with convolution. Since $CK\xi = \mathrm{Box}\,\xi$,

$$w[S_\varepsilon K\xi] = \mathrm{E}_0 \mathrm{Box}\, S_\varepsilon \xi = S_\varepsilon \xi.$$

The first-order scalar pullback is $\delta\varphi = \kappa\xi^\mu \partial_\mu \varphi$. The variation therefore is

$$\int f\Big[S_\varepsilon(\xi^\mu \partial_\mu \varphi) - (S_\varepsilon \xi)^\mu \partial_\mu S_\varepsilon \varphi\Big].$$

Take a compact cutoff equal to one on a neighbourhood of the ε-sampling region of $\mathrm{supp}\, f$, and set $\xi = t\partial_t$ there. The printed scalar is globally smooth and satisfies $\mathrm{Box}\,\varphi = 0$. Evenness and normalization give, on that neighbourhood,

$$S_\varepsilon t = t, \qquad S_\varepsilon(t^2) = t^2 + \varepsilon^2 m_{00}, \qquad \partial_t S_\varepsilon \varphi = t.$$

Substitution proves . A smooth nonnegative radial mollifier of integral one has $m_{00} > 0$.

Thus this route constructs positive-width free products but does not retain the unchanged classical gauge covariance. Quantum-order counterterms cannot cancel the displayed classical coefficient. The analogous distinction between convolution and a density ramp is explicit in Cheung's comparison track.

Route B1b: a divergence-form density preparation

Choose a smooth ramp

$$\sigma_\varepsilon(t) = \chi(t/\varepsilon), \qquad \chi = 0 \text{for} t \leq 0, \qquad \chi = 1 \text{for} t \geq 1, \qquad 0 \leq \chi \leq 1.$$

It is flat at the endpoints.

Define a coordinate density

$$A^{\alpha\beta}_\varepsilon[q] = \eta^{\alpha\beta} + \sigma_\varepsilon\big(\sqrt{-q}\, q^{\alpha\beta} - \eta^{\alpha\beta}\big).$$

The prepared coordinates solve

$$\partial_\alpha\big(A_\varepsilon^{\alpha\beta}[q]\partial_\beta \mathrm{X}^\mu_{q,\varepsilon}\big) = 0$$

with the same coordinate-time Cauchy data as .

The coefficients are

$$w_\varepsilon^\mu[h] = -E_0^{\mathrm{R}}\partial_\alpha\big(\sigma_\varepsilon a_1^{\alpha\mu}[h]\big),$$

$$v_\varepsilon^\mu[h,k] = -E_0^{\mathrm{R}}\Big[\partial_\alpha\big(\sigma_\varepsilon(a_1^{\alpha\mu}[k] + a_2^{\alpha\mu}[h])\big) + \partial_\alpha\big(\sigma_\varepsilon a_1^{\alpha\beta}[h]\partial_\beta w_\varepsilon^\mu[h]\big)\Big].$$

The resulting functional $\mathscr{O}^{\mathrm{D}}_{f,\varepsilon}$ is defined by formal inverse-coordinate composition, not by assigning a finite value to one graph.

Remark (Source smoothing is not density preparation). Already at first order,

$$w_\varepsilon^\mu[h] = E_0^{\mathrm{R}}\Big[\sigma_\varepsilon C^\mu h - (\partial_\alpha\sigma_\varepsilon)a_1^{\alpha\mu}[h]\Big].$$

The prescription $E_0^{\mathrm{R}}(\sigma_\varepsilon Ch)$ omits the second term. It is a different coordinate construction. The general density/source distinction is also the subject of XII-B.

A tree-composition lemma

For covectors compared in the fixed Minkowski trivialization, put

$$Z_n = \{(x_1,p_1;\dots;x_n,p_n) : p_1 + \cdots + p_n = 0\} \setminus 0, \qquad Z_1 = \varnothing.$$

A *balanced line* is a bidistribution with wavefront contained in $(x,p;y,-p)$. Derivatives of translation-invariant Green kernels and of $\delta(x-y)$ are examples.

Lemma (Compact balanced-tree composition). *Let a finite tree have internal and external vertices, one balanced line on each edge, and smooth vertex weights. Suppose the joint support of the weighted lines is compact. Then their product is defined, its pushforward over internal vertices is defined, and an n-slot output kernel has wavefront contained in* Z_n.

Differentiation and multiplication by fixed smooth coefficients preserve this conclusion.

Proof. Pull each line back to the full vertex configuration space. Its covector is p at one endpoint, $-p$ at the other, and zero elsewhere.

A forbidden product cancellation would assign line covectors whose signed sum vanishes at every vertex. In a forest a leaf forces the covector on its only incident edge to be zero. Removing that edge and repeating forces every line covector to be zero. This contradicts the nonzero covector required for a forbidden cancellation. Adding lines successively therefore defines the product.

The same leaf recursion bounds every line covector by a fixed multiple of the sum of the vertex covector norms. Thus the covector-sum upper bound is closed; no circulation with unbounded internal covectors is hidden in it.

Compact support makes projection over the internal variables proper. A pushforward wavefront point has zero covectors at all internal vertices. Summing the remaining vertex covectors cancels every line pair, so the external sum is zero. The distributional product, pullback, and pushforward criteria are the standard ones [9, 10].

The lemma is a tree statement. It does not define loops or the later grouped characteristic composition.

Theorem (Finite-width insertion admission). *For fixed $\varepsilon > 0$:*

1. *$\mathscr{O}^{\mathrm{D},(1)}_{f,\varepsilon}$ has a compact mixed kernel with wavefront in Z_2;*

2. *$\mathscr{O}^{\mathrm{D},(2)}_{f,\varepsilon}[h,0,\varphi]$ has a compact kernel with wavefront in Z_3;*

3. *the k-linear contribution is $\mathscr{O}^{\mathrm{D},(1)}_{f,\varepsilon}[k,\varphi]$;*

4. *these polynomials and all their fixed smooth directional derivatives belong to $\mathcal{P}_{\mathrm{mc,pol}}$. The coefficient maps are continuous on fixed-support test stages with their corresponding normal-topology kernel stages.*

Proof. At order one, integration by parts gives a derivative of

$$f(x)\partial E^{\mathrm{R}}_0(x-z)\sigma_\varepsilon(z)$$

with constant fibre tensors. It is a two-slot balanced kernel.

At order two, the inverse-coordinate formula contains:

1. two first-order coordinate lines ending at the observation vertex and a scalar derivative there;

2. a coordinate line from the observation to a local quadratic metric vertex, with two metric slots joined to that vertex by delta lines;

3. a nested coordinate line, a local metric slot, and a further first-order coordinate line. Every term is a finite sum of differentiated balanced trees. The *k*-term follows by linearity of a_1.

The observation vertex lies in supp *f*. Every retarded ancestor lies in its causal past, and each coordinate-source vertex has a factor σ_ε or a derivative of σ_ε; it is therefore at nonnegative time. Delta-attached field slots coincide with such vertices. All relevant supports lie in a fixed compact causal region. Lemma applies.

A nonzero tuple wholly in the future or past causal cone cannot have zero sum. Thus Z_n implies the microcausal exclusion. Smearing a slot against a smooth configuration inserts zero covector in that slot, so the remaining tuple still has zero sum.

For continuity, there are finitely many smooth multiplications, compatible fixed-kernel products, differentiations, and support-proper projections. Their normal-topology continuity and hypocontinuity properties apply on the just-checked stages [10]. No global completeness or global hypocontinuity claim is required.

Classical covariance of the prepared model

Proposition (Plateau covariance at fixed width). *Assume*

$$\overline{U} \subset \operatorname{int}\{g = 1\} \cap \{t > \varepsilon\}.$$

For the same restricted transformations as before,

$$\mathrm{X}_{F^* q,\varepsilon} = \mathrm{X}_{q,\varepsilon} \circ F \bmod \kappa^3,$$

and

$$\mathscr{O}^{\mathrm{D}}_{f,\varepsilon}[F^* q, F^* \varphi] = \mathscr{O}^{\mathrm{D}}_{f,\varepsilon}[q, \varphi] \bmod \kappa^3.$$

Proof. Write

$$A_\varepsilon[q] = (1 - \sigma_\varepsilon)A[\eta] + \sigma_\varepsilon A[q].$$

On the support of the transformation, $\sigma_\varepsilon = 1$, and outside that support the transformation and its derivative are the identity. Hence this density transforms naturally under the restricted pullback. The transformation is the identity near the initial surface, so Cauchy uniqueness proves the coordinate identity. Formal inversion and scalar pullback prove the observable identity.

This is the mechanism of Cheung's density-ramp classical construction. It proves a classical identity, not that the same gauge action is a derivation of the free Wick product.

The sharp limit changes the data

Theorem (First-order transmission law). *Let h be smooth near the closed future half-space. On every compact subset of* $t > 0$*,*

$$w_\varepsilon[h] \to w_{\mathrm{tr}}[h] \quad \text{in} C^\infty,$$

where

$$w^\mu_{\mathrm{tr}}[h] = \mathrm{E}_0 C^\mu h - \frac{\sin(t\sqrt{-\Delta})}{\sqrt{-\Delta}} a_1^{0\mu}[h](0,\cdot).$$

In particular,

$$w_{\mathrm{tr}}|_\Sigma = 0, \qquad \partial_t w^\mu_{\mathrm{tr}}|_{\Sigma^+} = -a_1^{0\mu}[h]|_\Sigma.$$

The prepared limit is generally not the sharp prescription $w = \mathrm{E}_0 Ch$*.*

Proof. Equation gives

$$w^\mu_\varepsilon = E_0^{\mathrm{R}}(\sigma_\varepsilon C^\mu h) - E_0^{\mathrm{R}}(\sigma_\varepsilon' a_1^{0\mu}[h]).$$

On bounded causal regions, $\sigma_\varepsilon Ch \to \theta Ch$. The first response converges to $\mathrm{E}_0 Ch$ away from the initial surface.

For the second response use the smooth Cauchy formula

$$E_0^{\mathrm{R}}(\sigma_\varepsilon' a_1^{0\mu}[h])(t) = \int_0^\varepsilon \frac{\sin((t-s)\sqrt{-\Delta})}{\sqrt{-\Delta}}\,\sigma_\varepsilon'(s) a_1^{0\mu}[h](s)\,\mathrm{d}s.$$

The positive smooth weight σ_ε' has integral one. For t bounded away from zero, continuous smooth Cauchy dependence, with spatial localization on the fixed causal compact, permits passage to the limit in every derivative. The limit is the homogeneous solution displayed in . A Taylor estimate gives an $O(\varepsilon)$ rate on each such smooth seminorm, with a constant depending on finitely many derivatives of h.

That homogeneous solution has initial displacement zero and initial velocity $a_1^{0\mu}[h](0)$.

The sharp-limit comparison is already decided at first order. No assertion about a second-order limit can make the nonzero initial velocity disappear. This is a genuine model change, not a renormalization convention.

Finite-width free products and ordering

Use the even field pair (h, φ) and the fixed Hermitian two-point matrix

$$H_{hh\,\alpha\beta\rho\sigma} = P_{\alpha\beta\rho\sigma}\Delta_0^+, \qquad H_{\varphi\varphi} = \Delta_m^+, \qquad H_{h\varphi} = 0,$$

with

$$P_{\alpha\beta\rho\sigma} = \eta_{\alpha(\rho}\eta_{\sigma)\beta} - \tfrac{1}{2}\eta_{\alpha\beta}\eta_{\rho\sigma}.$$

The scalar mass may be any $m \geq 0$. No positivity is assumed for the graviton block.

For finite polynomials define

$$F \star_H G = m\exp(\hbar\Gamma_H)(F \otimes G).$$

The contraction series terminates for each polynomial pair.

Theorem (The finite-width free algebra). *At fixed preparation width, the polynomials of Theorem , the positive-width convolution polynomials, ordinary compact local polynomial fields and vertices, and their fixed*

*smooth directional derivatives generate a unital associative free *-algebra in* $\mathcal{P}_{\mathrm{mc,\,pol}}[[\hbar,\kappa]]$.

The adjoint is complex conjugation of symbols:

$$(F \star_H G)^* = G^* \star_H F^*.$$

The quotient by (κ^3) *is a consistent coordinate-order truncation. No field-degree truncation is imposed.*

Proof. The admitted finite kernels obey the microcausal exclusion. For each contraction, a forbidden cancellation against the oriented Hadamard lines would force one input covector tuple wholly into one of the excluded causal cones. Thus the products and proper contractions are defined and their output kernels remain compact and microcausal.

On three factors, the constant even contraction operators $\Gamma_{12}, \Gamma_{13}, \Gamma_{23}$ commute. Both associations therefore equal

$$m^{(3)} \exp\left[\hbar(\Gamma_{12} + \Gamma_{13} + \Gamma_{23})\right].$$

The same microlocal argument justifies the finite multicontractions. This is the polynomial Wick construction [11, 13, 12].

Hermiticity of H proves the adjoint identity by reversing the contraction slots. Constants are the unit. Products can increase field degree, but each coefficient is still a finite polynomial. The formal parameter ideal (κ^3), unlike an arbitrary field-degree cutoff, is stable under multiplication.

This theorem closes Level II for a *prepared free* insertion through second coordinate order. It does not close Level III.

Proposition (Ordering transport of the generated sector). *Let* $H' = H + d$*, where* d *is smooth, symmetric, and compatible with the declared reality convention. Define*

$$\alpha_d = \exp\left(\frac{\hbar}{2}\left\langle d, \frac{\delta^2}{\delta\Phi^2}\right\rangle\right).$$

Then

$$\alpha_d(F \star_H G) = \alpha_d F \star_{H'} \alpha_d G.$$

For the prepared first and second coordinate coefficients,

$$\alpha_d \mathscr{O}_\varepsilon^{(1)} = \mathscr{O}_\varepsilon^{(1)} + \hbar c_d, \qquad \alpha_d \mathscr{O}_\varepsilon^{(2)}[h, 0, \varphi] = \mathscr{O}_\varepsilon^{(2)}[h, 0, \varphi] + \hbar L_d,$$

where c_d is a scalar and L_d is a compact smooth linear field smearing.

Consequently the generated sector, including ordinary local polynomials and smooth directional descendants, is transported onto the corresponding sector generated by the unchanged prepared formulas.

Proof. The self-contraction generator obeys

$$D_d m = m(D_d^{(1)} + D_d^{(2)} + \hbar \Gamma_d).$$

Exponentiating the commuting operators proves the intertwining identity and gives inverse α_{-d}.

A quadratic self-contraction is a scalar. A cubic has only a linear correction. For a projected linear kernel to have a nonzero wavefront covector p, the original Z_3-bounded kernel would need a tuple $(0, 0, p)$. The zero-sum condition forces $p = 0$. Thus the correction is smooth; compactness follows from proper projection.

Smooth directional differentiation commutes with the self-contraction operator. Local polynomial ordering corrections are again local polynomials with smooth coefficients. These and compact linear fields are already among the generators. The inverse transport gives the reverse inclusion.

Ordering transport compares algebra representations. A state must be transported too; keeping a changed symbol and an unchanged numerical state functional is not physical matching. The related distinction is explicit in XII-A and VII-H [7].

Route B2: a minimal boundary-sensitive classical carrier

A classical boundary carrier can be constructed without asserting a quantum multiplication.

Definition (Minimal actual-functional carrier). Let $\mathcal{P}_{\Sigma,\min}$ be the algebra of actual finite polynomial functionals

generated, under ordinary pointwise multiplication, complex conjugation, and all fixed smooth directional derivatives, by:

1. $\mathcal{P}_{\mathrm{mc,pol}}$;

2. all sharp $\mathscr{B}_{f}$;

3. all linear sharp coordinate functionals ℓ_{a}. Identical functionals on smooth configurations are identified.

The pointwise products are multilocal polynomials. They are not declared local merely because their factors were local.

A well-defined boundary charge

At a fixed configuration Φ, a metric derivative of an element of $\mathcal{P}_{\Sigma,\min}$ has the form

$$F_{h}^{'}(\Phi) = u_{\Phi} + \theta r_{\Phi} + \delta(t) v_{\Phi},$$

where u_{Φ} is a compact distribution with no timelike wavefront covectors and r_{Φ}, v_{Φ} are smooth near Σ. The boundary coefficient v_{Φ} is spatially compact.

Lemma (Intrinsic charge). *The coefficient v_{Φ} in is unique. Write it as* $\mathrm{ch}_{\Phi}(F)$. *It is complex linear and satisfies*

$$\mathrm{ch}_{\Phi}(FG) = F(\Phi)\mathrm{ch}_{\Phi}(G) + G(\Phi)\mathrm{ch}_{\Phi}(F).$$

Every microcausal polynomial has zero charge.

Proof. The decomposition holds for the generators by Proposition . A smooth metric directional derivative preserves the same form, and a scalar directional derivative changes only the smooth source in that formula. The ordinary Leibniz rule preserves the decomposition under products.

If two decompositions existed, their difference would express $\delta(t)(v - v^{'}) + \theta(r - r^{'})$ as a distribution without timelike wavefront covectors. A nonzero $v - v^{'}$ produces the normal Fourier order-zero term of Proposition ; neither the step term nor a distribution microlocally smooth in the normal directions can cancel it. Hence $v = v^{'}$.

Uniqueness makes the charge intrinsic to the actual functional and proves linearity and . For a microcausal polynomial its first derivative has no timelike covectors, so its charge is zero.

Topology

On each fixed-support, bounded-polynomial-degree stage, give $\mathcal{P}_{\Sigma,\min}$ the locally convex topology generated by:

1. the strong distribution seminorms of its symmetric coefficient kernels;

2. every scalar pairing

$$F\text{longmapsto}\left\langle \mathrm{ch}_{\Phi}(D_{\chi_1}\cdots D_{\chi_r}F),\zeta\right\rangle,$$

for fixed smooth directions χ_j, fixed smooth configuration Φ, and compact spatial boundary test ζ. Assemble the stages by the locally convex inductive-limit topology. Denote it by τ_{bd}.

Theorem (Boundary carrier and nonconvergence). *The space* $(\mathcal{P}_{\Sigma,\min},\tau_{\mathrm{bd}})$ *is Hausdorff. Fixed smooth directional differentiation and conjugation are continuous. Pointwise multiplication is separately continuous. Bulk inclusion is injective and continuous on fixed-support, fixed-wavefront normal-topology stages.*

For every nonzero real f, the unchanged regular approximants

$$\mathscr{B}_{f,\varepsilon}=\mathscr{B}_f[S_\varepsilon h,S_\varepsilon\varphi]$$

do not converge to $\mathscr{B}_f$ *in* τ_{bd}. *No quantum product is included in this theorem.*

Proof. Symmetric coefficient kernels determine an actual polynomial uniquely by polarization. Their continuous map into the Hausdorff distribution spaces therefore separates points.

Differentiation contracts coefficient kernels with fixed smooth directions, and sends each charge seminorm to another seminorm in the defining list. Conjugation has the same property.

On a fixed stage, multiplication by a fixed polynomial uses finite tensor products with fixed compact distributions and finite symmetrization. These are continuous on the relevant strong fixed-support spaces. Charges of every directional derivative of a product are finite Leibniz sums of evaluations and charges, which are continuous there.

The inductive-limit universal property gives global separate continuity.

The normal topology includes the strong distribution seminorms. Bulk charges vanish, so fixed bulk-stage inclusions are continuous. They are injective because actual functionals are not quotiented further.

Use the witness configuration of Theorem . The charge of $\mathscr{B}_f$ is a nonzero smooth boundary tensor. Choose a compact boundary test detecting it. Every $\mathscr{B}_{f,\varepsilon}$ has a smooth kernel, hence zero charge at every configuration. The continuous charge pairing separates the proposed limit from all approximants.

The construction is a minimal comparison carrier, not an identification with every stage of IX-A. Its mechanism agrees with the boundary-sensitive nonconvergence explicitly recorded there. It changes the convergence hypothesis used in VII-C; it does not construct the missing sharp multiplication on the IX carrier.

The triangle obstruction and what VII-D does supply

This section uses one explicitly stated numerical input from the supplied VII-C/D sources. Its proof is not replaced by a citation to a general extension theorem.

Signature matching

VII-C/D use $\eta_{\mathrm{C}} = -\eta$ and $\mathrm{Box}_{\mathrm{C}} = -\mathrm{Box}$. The real field identification

$$h_{\mathrm{C}} = -h, \qquad \varphi_{\mathrm{C}} = \varphi$$

gives

$$\mathrm{E}_0^{\mathrm{C}} C_{\mathrm{C}} h_{\mathrm{C}} = \mathrm{E}_0 C h.$$

Their sharp insertion therefore pulls back to our $\mathscr{B}_f$. Their middle observable $h^2_{00,\mathrm{C}}$ pulls back to h^2_{00}. The graviton two-point tensor has two metric factors, so its displayed numerical tensor is unchanged. Both insertion signs in the triple occur twice.

Input theorem (The complete numerical VII-C/D family). Let

$$W_g = \int g(y) h_{00}(y)^2 \, \mathrm{d}^4 y, \qquad T_\varepsilon^\rho = \mathrm{ev}_0 [\hbar^3] \big(\mathscr{B}_{f,\varepsilon} \star_H W_g \star_H \mathscr{B}_{f',\varepsilon} \big)$$

be understood first on product tests and then as the supplied joint distribution on $X = (M_+^\circ)^3$. The field mollifier is used on both slots of both insertions; the middle observable is unsmeared.

There are a nonzero positive residue r and a common distribution U_m such that

$$T_\varepsilon^\rho = \log(\varepsilon^{-1}) r + U_m + \kappa_\rho r + O_{\mathscr{D}'}(\varepsilon(1 + |\log \varepsilon|)).$$

On each fixed compact, the error is bounded by the stated factor times a test seminorm of order 64. Specifically,

$$\mathrm{pair}\, rf \otimes g \otimes f' = \frac{1}{64(2\pi)^3} \int g(y) \int_{S^2} A_f(y, \mathbf{n}) A_{f'}(y, \mathbf{n}) \, \mathrm{d}\Omega(\mathbf{n}) \, \mathrm{d}^4 y,$$

$$A_f(y, \mathbf{n}) = \int f(t, \mathbf{y} + (t + y^0)\mathbf{n}) \, \mathrm{d}t.$$

There are nonnegative pairwise-disjoint f, g, f' for which this pairing is positive.

For a subtraction scale $\mu > 0$,

$$u_\mu^\rho = U_m + (\kappa_\rho + \log \mu) r.$$

For any two allowed fixed radial profiles,

$$u_\mu^a - u_\mu^b = c_{ab} r, \qquad c_{ab} = \int_0^\infty \frac{\theta_a(u)^4 - \theta_b(u)^4}{u} \mathrm{d}u \in \mathbb{R}.$$

These are numerical distributional statements, including strong convergence and the full reflected-diagonal intersection [3, 4].

Only this declared input is needed for the following theorem.

Theorem (Sharp Wick-extension incompatibility through $\hbar^3$). *There is no Hausdorff locally convex algebra over $\mathbb{C}[\hbar]/(\hbar^4)$ satisfying all of:*

1. *the regular Wick products of the displayed approximants and W_g agree with the old products through $\hbar^3$;*

2. *each unchanged $\mathscr{B}_{f,\varepsilon}$ converges to an admitted sharp $\mathscr{B}_f$;*

3. *multiplication is hypocontinuous with respect to bounded sets;*

4. *extraction of the field-independent $\hbar^3$ coefficient is continuous and has finite complex values. For a separated positive-residue test with coefficient $C > 0$, the violating sequence has the quantitative lower bound*

$$\operatorname{Re} T_{\varepsilon}^{\rho} \geq C \log(\varepsilon^{-1}) - K$$

for all sufficiently small ε.

Proof. Choose the separated tests from Input theorem . The approximant families converge by (Q2), hence are bounded. Hypocontinuity implies separate continuity and continuity along pairs of convergent sequences: split the difference of two products and use uniform continuity over the bounded family in the other slot.

Thus

$$(\mathscr{B}_{f,\varepsilon} \star W_g) \star \mathscr{B}_{f,\varepsilon}$$

converges. By (Q4), its displayed coefficient converges to a finite number.

By (Q1) it is the old numerical coefficient. Equation instead gives the stated logarithmic lower bound and divergence. This is a contradiction.

No adjoint or Ward identity can restore all four requirements: the contradiction occurs before either additional condition is used. The boundary topology of Theorem avoids (Q2), exactly as declared.

Proposition (Why the finite part is not this extension). *The VII-D finite part cannot be the continuous value of the old triple under (Q1)–(Q4). A counterterm supported only on the ordinary observation diagonals cannot cancel its logarithmic residue on all tests.*

Proof. Assigning the finite part only at the sharp endpoint leaves the divergent regular sequence unchanged. Subtracting it along that sequence changes the already prescribed products in (Q1).

For the second assertion, every ordinary-diagonal-supported distribution pairs to zero with the pairwise-disjoint test bank of , while *r* has positive pairing there.

The operator monomial at this graph order is the identity: the three quadratic insertions have six legs, and the reflected triangle contracts all six. This identifies the numerical coefficient's place in the old word. It does not construct the other output coefficients or the next binary multiplication.

IX-B explicitly preserves this non-factorization boundary.

Necessary compatibility beyond one finite graph

Proposition (Associativity and normalization interfaces). *Suppose a proposed sharp product has defined bilinear maps*

$$F \star G = \sum_{j=0}^{3} \hbar^{j} \mu_{j}(F, G) \bmod \hbar^{4}, \qquad \mu_{0}(F, G) = FG.$$

Then associativity is equivalent to

$$\delta\mu_{1} = 0, \qquad \delta\mu_{n}(F, G, K) = \sum_{\text{substack } i+j=n}$$

where

$$\delta\mu(F, G, K) = F\mu(G, K) - \mu(FG, K) + \mu(F, GK) - \mu(F, G)K.$$

If two products agree below order p, a comparison $Z = \mathrm{id} + \hbar^{p} D \bmod \hbar^{p+1}$ *must satisfy*

$$\mu_{p}^{b}(F, G) - \mu_{p}^{a}(F, G) = D(FG) - D(F)G - FD(G).$$

Proof. Expand both associations coefficientwise and collect all terms with one occurrence of μ_{n}; the remaining terms are the right-hand side of . Expanding $Z(F \star_{a} G) = ZF \star_{b} ZG$ at order p gives .

These equations apply only when their maps and compositions are defined. The scalar identity $u_{\mu}^{a} - u_{\mu}^{b} = c_{ab} r$ neither defines D nor verifies on the generated sector.

A derivative-order bound also need not make the normalization space finite-dimensional: arbitrary smooth tangential coefficients multiplying a normal delta already give an infinite-dimensional space. This distinction is central in the supplied XV-F ambiguity discussion.

Route B3: constrained auxiliary coordinates

Classical equivalence, not an independent coordinate field

Introduce auxiliary smooth variables w^μ, v^μ, and impose

$$\mathrm{Box}\, w^\mu = C^\mu h,$$

$$\mathrm{Box}\, v^\mu = C^\mu k - \partial_\alpha a_2^{\alpha\mu}[h] - \partial_\alpha (a_1^{\alpha\beta}[h]\partial_\beta w^\mu),$$

$$w|_\Sigma = \partial_t w|_\Sigma = v|_\Sigma = \partial_t v|_\Sigma = 0.$$

Theorem (Classical auxiliary-coordinate equivalence). *For every smooth metric jet* (h, k)*, the constraints have exactly one solution. Projection from their solution graph to* (h, k) *is bijective, and substituting that solution into*

$$\int f\Big[\varphi - \kappa w^\mu \partial_\mu \varphi + \kappa^2 \Big((w^\nu \partial_\nu w^\mu - v^\mu)\partial_\mu \varphi + \tfrac{1}{2} w^\mu w^\nu \partial_\mu \partial_\nu \varphi\Big)\Big]$$

gives the original sharp $\mathscr{O}_f$*.*

The same statement holds for the prepared density equations, with their corresponding coefficients and zero data.

Proof. The first equation is a linear Cauchy problem with prescribed zero data. Its unique solution is $w = \mathrm{E}_0 C h$. With that solution inserted, the second is another linear inhomogeneous Cauchy problem, whose unique solution is . The observable equality is substitution into its formal inverse-map expansion. The prepared equations are handled in the same order.

Without the data in , arbitrary homogeneous coordinate fields remain. Quantizing those fields would not be a quantization of the fixed Cauchy prescription.

Necessary quantum contractions

At first order, write a linear auxiliary constraint as

$$\mathscr{C}_a = w(a) - h(L^* a).$$

The second term is the actual sharp distributional smearing. Its use below is a constraint test, not an assertion that

it already lies in the old microcausal algebra.

Proposition (Contractions forced by exact quantum elimination). *Suppose an auxiliary Wick construction is defined on the displayed linear constraints and preserves their ordinary graph ideal as a quantum ideal, with no nonzero scalar identified with zero. Then its contractions must satisfy*

$$H_{wh} = LH_{hh}, \qquad H_{hw} = H_{hh}L^{\mathrm{T}}, \qquad H_{ww} = LH_{hh}L^{\mathrm{T}}$$

as identities of their admitted smeared pairings.

For the sharp zero-Cauchy prescription and the fixed de Donder matrix,

$$H_{ww} = 0, \qquad H_{wh} \neq 0.$$

Consequently an independent central w-sector cannot implement exact elimination while retaining the old hh contractions.

Proof. For a linear field $h(b)$,

$$\mathscr{C}_a \star h(b) = \mathscr{C}_a h(b) + \hbar H(\mathscr{C}_a, h(b)).$$

The first term belongs to the ordinary graph ideal. If that ideal is also a quantum ideal, the scalar contraction must vanish. This gives the first identity in . Reverse products give the transposed identity. Applying the same test to an auxiliary linear field gives the last identity.

The tensor contraction $C_x H_{hh} C_y^{\mathrm{T}}$ is a constant multiple of $\eta^{\mu\nu}\mathrm{Box}\,\Delta_0^+$, hence zero before the two Cauchy solution maps are applied. This yields $H_{ww} = 0$.

The mixed component is

$$H^0_{wh00}(x,y) = \mathrm{E}_0^x \partial_t \Delta_0^+(x-y).$$

In the present $+---$ convention its mode formula is

$$\int \frac{\mathrm{d}^3 p}{(2\pi)^3} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \left[\left(\frac{t}{4P} - \frac{i}{8P^2} \right) e^{-iP(t-s)} + \frac{i}{8P^2} e^{iP(t+s)} \right], \qquad P = |\mathbf{p}|.$$

It is nonzero and has the reflected branch. Thus assigning zero mixed contractions to an independent central coordinate would violate the first identity.

This does not exclude every quantum-corrected constraint formulation. It excludes the stated exact graph-ideal descent with independent central-coordinate contractions.

With the forced cross-contractions, the sharp composite word inherits the VII-C triangle. Auxiliary notation does not remove Theorem .

A genuine linear free-shell quantum repair

Lemma (Smooth causal propagation of the linear boundary kernel). *For $q_a = L^* a$, the causal solution $u_a = E_0 q_a$ is smooth and spacelike compact.*

Proof. Proposition gives $\mathrm{WF}(q_a) \subset N^*\Sigma$, consisting of timelike covectors. The causal propagator has only paired nonzero characteristic covectors, which are null. Its kernel-composition relation cannot transport a timelike input singularity. Thus the output has empty wavefront set. Its support is contained in $J(\operatorname{supp} q_a)$, giving spacelike compactness [8, 9].

Fix a smooth time cutoff ρ_0, zero sufficiently early and one sufficiently late, and put

$$q_{\mathrm{sm},a} = [\mathrm{Box}, \rho_0] u_a.$$

Theorem (Linear sharp coordinates in the free quantum algebra). *The tensor $q_{\mathrm{sm},a}$ is smooth and compact, depends linearly on a, and satisfies*

$$q_a - q_{\mathrm{sm},a} = \mathrm{Box}\, r_a$$

for a compact distribution r_a. Therefore

$$\ell_a[h] = h(q_{\mathrm{sm},a}) \quad \text{for every smooth} \mathrm{Box}\, h = 0.$$

The assignment

$$\hat{\ell}_a = \hat{h}(q_{\mathrm{sm},a})$$

is a well-defined linear element of the ordinary free on-shell field algebra. Finite products, adjoints, and their free contractions exist. Its class is independent of the chosen time cutoff.

Proof. Smoothness follows from Lemma . The commutator with a time cutoff is supported in a bounded time band. Spacelike compactness makes its support compact.

The retarded and advanced identities are

$$E_0^{\mathrm{R}} q_{\mathrm{sm},a} = \rho_0 u_a, \qquad E_0^{\mathrm{A}} q_{\mathrm{sm},a} = (\rho_0 - 1) u_a.$$

Hence $E_0(q_a - q_{\mathrm{sm},a}) = 0$. The retarded and advanced solutions of that compact source agree, so

$$r_a = E_0^{\mathrm{R}}(q_a - q_{\mathrm{sm},a}) = E_0^{\mathrm{A}}(q_a - q_{\mathrm{sm},a})$$

has compact support and $\mathrm{Box}\, r_a = q_a - q_{\mathrm{sm},a}$. Pair with a smooth homogeneous field to obtain the classical shell equality.

The smooth representative is an ordinary compact field test, so the usual free on-shell algebra admits it. Two choices have a smooth compact difference d with $E_0 d = 0$. Then $d = \mathrm{Box}\,(E_0^{\mathrm{R}} d)$, with $E_0^{\mathrm{R}} d$ smooth compact. The free equation ideal and the bisolution contraction kill that difference.

This is the linear repair of VII-B, now with its free algebra consequence made explicit [2]. It changes off-shell equality to free-shell equality.

For the bilinear insertion the test $a = f\, \mathrm{d}\varphi$ depends on the quantum scalar field. Applying a linear repair separately at each fixed φ does not prove a joint microcausal kernel, compatibility with contractions, or interacting equation-of-motion identities. Those obligations remain.

Presentation and restricted gauge complexes

Presentation descent

Proposition (Actual-functional descent). *Let* $\mathrm{Rec} : \mathcal{P} \to \mathcal{K}$ *be a surjective linear presentation map. A proposed linear construction* $\widetilde{M} : \mathcal{P} \to Y$ *descends to* $\mathcal{K}$ *if and only if*

$$\ker \mathrm{Rec} \subseteq \ker \widetilde{M}.$$

With the quotient topology, a continuous such map descends continuously.

Proof. Necessity follows by testing a null presentation. Conversely, two presentations of the same element differ by a reconstruction-null element, so their images agree. Surjectivity gives uniqueness, and the quotient topology gives continuity.

For binary constructions the same criterion is required in each slot. The ideal of presentation relations must also be stable under the proposed products.

The boundary and auxiliary identities in this paper are identities of actual classical functionals. Our finite-width free products are evaluated on their admitted kernels, so they respect those off-shell identities. A sharp graph extension specified on presentations must separately pass Proposition .

For example,

$$q\delta'(q) + \delta(q) = 0$$

is a null presentation with two nonzero summands. Selecting a highest presentation degree is not a construction on the resulting zero distribution. Cheung's XIV-J makes this exact test in its internal amplitude module.

A relative classical differential

Introduce an odd compact ghost $c \in C_c^\infty(U; TM)$ in finite Grassmann evaluations. On formal fields set

$$s_{\mathrm{rel}} q = \kappa \mathscr{L}_c q, \qquad s_{\mathrm{rel}} \varphi = \kappa c^\mu \partial_\mu \varphi, \qquad s_{\mathrm{rel}} c^\mu = \kappa c^\nu \partial_\nu c^\mu.$$

Use graded left differentiation. The last sign belongs to this explicitly declared convention.

Proposition (Relative classical closure). *The rule is nilpotent and tangent to the declared compact ghost space. For the sharp classical construction, and for the prepared density construction when* $U \subset \{t > \varepsilon\}$,

$$s_{\mathrm{rel}}\mathscr{O}_f = 0 \bmod \kappa^3.$$

The auxiliary constraints are preserved by the corresponding coordinate pullback law.

Proof. The support of $c^\nu \partial_\nu c^\mu$ remains compact in U. For a scalar,

$$s_{\mathrm{rel}}^2 \varphi = \kappa(s_{\mathrm{rel}} c^\mu)\partial_\mu \varphi - \kappa c^\mu \partial_\mu (s_{\mathrm{rel}}\varphi) = 0 :$$

the first-derivative terms cancel and the second-derivative term vanishes by odd antisymmetry. The tensor calculation is the same identity $\mathscr{L}_c^2 = \frac{1}{2}\mathscr{L}_{[c,c]}$, with $[c,c]^\mu = 2c^\nu \partial_\nu c^\mu$. It proves nilpotence on q and c.

Cauchy uniqueness gives $s_{\mathrm{rel}} \mathrm{X}^\mu = \kappa c^\nu \partial_\nu \mathrm{X}^\mu$. The inverse-coordinate and scalar pullbacks therefore cancel in the relational composition. This also preserves the coordinate equations and their fixed data, proving the auxiliary statement.

This is a relative classical Chevalley–Eilenberg structure. It is not a gauge-fixed quantum BV complex.

Proposition (Compact relative ghosts are not free ghost solutions). *A smooth spacetime-compact ghost supported strictly above* Σ *and satisfying* $\mathrm{Box}\, c = 0$ *is zero. Thus replacing the relative ghosts of by nonzero homogeneous free ghosts is not an identification of domains.*

Proof. The compact future support gives zero Cauchy data on Σ. Homogeneous Cauchy uniqueness gives $c = 0$.

The full observable can be classically closed while an individual geometric coefficient is not. In particular, $s_{\mathrm{rel}}\varphi$ contains the order-κ term that cancels the variation of the first dressing. Deleting it and calling the remainder a quantum Ward identity would change the rule.

Why quantum closure is still a separate gate

Even for the admitted prepared free polynomials, the classical rule s_{rel} has not been proved a derivation of $\star_H$. Its compatibility would require contraction identities for the fixed covariance and the nonlinear rule, followed by the

appropriate renormalized Ward identities for mixed products.

For a constructed quantum differential

$$\hat{s} = s_{\mathrm{cl}} + \sum_{j \geq 1} \hbar^{j} s_{j},$$

recursive insertion closure would require, at order n,

$$s_{\mathrm{cl}} \mathcal{O}^{[n]} = -\sum_{j=1}^{n} s_{j} \mathcal{O}^{[n-j]}.$$

The right-hand side is not an anomaly calculation until all its maps and products are defined. Bulk and initial-surface pieces require an actual support analysis before a decomposition into anomaly sectors is asserted.

Renormalized BV and multiple-insertion Ward identities supply the general setting for these questions [15, 16, 17]. They do not supply the missing maps for the sharp Cauchy insertion.

Interactions, normalization, and a controlled diagnostic

The full second-order bookkeeping

If the necessary retarded products are constructed, then

$$[\kappa^{2}] R_{V}(\mathcal{O}_{f}) = \mathcal{O}_{f}^{(2)} + R_{1}(V_{1}; \mathcal{O}_{f}^{(1)}) + R_{1}(V_{2}; \mathcal{O}_{f}^{(0)}) + \tfrac{1}{2} R_{2}(V_{1}, V_{1}; \mathcal{O}_{f}^{(0)}).$$

The bare $\mathcal{O}_{f}^{(2)}$ contains no separately solved interacting scalar coefficients. A second metric coefficient contributes through its affine k-slot once.

Local interaction vertices belong to the ordinary polynomial domain, but mixed time-ordered products with a nonlocal prepared or sharp insertion require their own construction. The Bogoliubov formula is applicable only after those time-ordered products exist [13, 14]. Free Wick closure is not that existence theorem.

Functional support reaches into D_{f}, including the initial surface for the sharp object. Causal factorization cannot be based only on $\operatorname{supp} f$.

Normalization categories

A future interacting comparison must distinguish:

$$\mathfrak{N}_{\mathrm{bulk}},\quad \mathfrak{N}_{\Sigma},\quad \mathfrak{N}_{\mathrm{insertion}},\quad \mathfrak{N}_{\mathrm{mixed}}.$$

These spaces have not been classified for the unresolved sharp construction.

The only complete numerical catalogue imported here is $\mathbb{R}r$ for the parent VII-D family. The class $\mathbb{R}\hbar\delta_{0}$ of VII-H is the different free Wick-square response benchmark [7]. Neither catalogue is a normalization theorem for the gravitational characteristic coefficient.

Changing the initial coordinate data, omitting the layer source in , or replacing the transmission law changes the observable. Such changes are not automatically finite quantum normalizations.

A finite-width algebra-level coefficient

Let $h(a)$ and $\varphi(u)$ be compact smooth linear fields. For the density-prepared first dressing,

$$B^{\mathrm{D}}_{f,\varepsilon} = -\int f\, w^{\mu}_{\varepsilon}[h]\partial_{\mu}\varphi.$$

The free Gaussian coefficient functional $\omega_{H}(F) = F(0)$ gives the exact finite expression

$$\omega_{H}\Big(B^{\mathrm{D}}_{f,\varepsilon} \star_{H} h(a) \star_{H} \varphi(u)\Big) = -\hbar^{2}\int f\, w^{\mu}_{\varepsilon}[H_{hh}a]\,\partial_{\mu}(H_{\varphi\varphi}u)\,\mathrm{d}^{4}x.$$

There is one allowed complete contraction: the metric slot meets $h(a)$, and the scalar slot meets $\varphi(u)$.

Proposition (Scope of the finite coefficient). *Equation is a defined coefficient of the constructed finite-width free algebra, independent of its inverse-coordinate or integrated-kernel presentation. It does not constitute a positive state on the full indefinite gravitational algebra.*

Proof. Each propagated smooth test is smooth on the compact observation region, and the integral is finite. The finite-width product theorem justifies the contractions. Actual-functional presentation equality and linearity give presentation independence.

The tensor P is indefinite, so positivity of $\omega_H(F^* \star_H F)$ for arbitrary graviton polynomials has not been asserted and does not follow from scalar vacuum positivity.

A positive scalar diagnostic with a fixed classical metric

Fix a smooth classical metric jet (h, k), a finite width, and retain only the scalar as a quantum field. The prepared relational coefficient is scalar-linear, so there is a compact smooth test

$$d_\kappa = f + \kappa d_1 + \kappa^2 d_2$$

such that

$$\mathscr{O}^{\mathrm{D}}_{f,\varepsilon}[h, k, \varphi] = \varphi(d_\kappa) \bmod \kappa^3.$$

Smoothness follows either by transposition through the smooth classical coefficients or by the one-external-slot case of Lemma .

Proposition (Positive marginal state evaluation). *In the Minkowski scalar vacuum,*

$$\omega_0\big(\varphi(d_\kappa)^* \star \varphi(d_\kappa)\big) = \hbar \int_{\mathbb{R}^3} \frac{|\widehat{d}_\kappa(\omega_m(\mathbf{p}), \mathbf{p})|^2}{(2\pi)^3 2\omega_m(\mathbf{p})} \mathrm{d}^3\mathbf{p}.$$

This is finite coefficientwise. There are real compact f for which its leading coefficient is strictly positive. It is a scalar marginal diagnostic on a fixed classical metric, not a state evaluation of an interacting quantum metric observable.

Proof. The scalar two-point spectral formula gives . Smooth compact smearings have rapidly decreasing Fourier transforms, and the massless infrared factor is locally integrable in three dimensions.

For example, at $m = 0$, take a real nonnegative compact bump f with nonzero integral. Its Fourier transform is nonzero near zero, including a nonempty portion of the positive mass shell. The leading spectral integral is therefore strictly positive.

This provides one controlled state calculation without asserting a physical state on the indefinite graviton sector or

formal positivity of a numerical perturbative truncation.

Later boundaries and the exact route decisions

What the later corpus does and does not change

The supplied IX-A carrier is classical, and IX-B explicitly leaves second sharp multiplication open. The supplied IX-E material also reports a relative partial free construction through $\hbar^3$ on its named domain; it is therefore too strong to say that no partial associativity result exists. What remains absent there is the required sharp time-ordered and retarded construction.

The supplied XII-D admits a changed completed classical insertion. XII-F separately proves that the auxiliary remainder need not vanish on the gauge-fixed free shell. Admission is not that invariance theorem.

For the later analytic chain, XIV-H leaves an internal transition obligation open. XIV-I then supplies its named I2 amplitude construction, but not the I3 joint map. XIV-J supplies its supported-sector reference away from the characteristic locus, but not the full J2 map. An earlier H2 stopping statement must not be repeated as if that subsequent I2 progress had not occurred.

The supplied XV interfaces and XVI-N/O statements do not admit the inherited interacting quantum coefficient. Their relative identities and kernel results are not substitutes for the missing full composition or insertion Ward identity.

VII-E's conditional distributional dictionary and VII-G's finite instruments remain observation and implementation tools, not constructors of a missing graph [5, 6].

All repair routes, with separate verdicts

toprule Route Positive result Exact limitation

midrule B1a: field convolution Regular free algebra at positive width; classical smooth-field convergence Explicit classical covariance defect ; no unchanged gauge-invariant repair

B1b: density preparation Microcausal insertion through coordinate order two; associative free sector; classical plateau covariance Sharp limit has the transmission data ; not the original observable

B2: boundary carrier Exact sharp classical admission, intrinsic boundary charge, Hausdorff topology No sharp quantum product; unchanged regular approximants do not converge in this topology

B2: normal-topology Wick extension Exact identification of the incompatible requirements (Q1)–(Q4) cannot coexist through $\hbar^3$

B3: constrained coordinates Classical equivalence and necessary quantum cross-contractions Independent central coordinates fail exact elimination; nonlinear renormalized descent unresolved

Linear free-shell route Sharp linear coordinate classes in the ordinary free quantum algebra Does not prove a joint bilinear or interacting repair

bottomrule

Theorem (VII-I release). *For the declared Cauchy prescription and restricted gauge domain:*

1. *the original nonzero-smearing first dressing cannot have an off-shell-equal classical symbol in the microcausal polynomial algebra;*

2. *a positive-width density-prepared replacement has admitted compact polynomial kernels through second coordinate order and compatible associative free products, adjoint, and ordering transport;*

3. *convolution and density preparation are not interchangeable: the former has the explicit covariance defect, while the latter has the explicit changed sharp transmission data;*

4. *an exact sharp classical boundary carrier and classical auxiliary-coordinate formulation exist at their stated levels;*

5. *unchanged sharp admission, old-product agreement, bounded-set hypocontinuity, and continuous triangle extraction cannot coexist;*

6. *linear sharp coordinates admit exact free-shell quantum representatives;*

7. *no nonlinear sharp interacting gauge-cohomology class, complete sharp normalization theorem, or full physical gravitational correlation is constructed. The specified incompatibilities are proved obstructions. The finite-width free construction is a controlled repaired construction. The unrestricted sharp interacting target outside those incompatible requirements remains unresolved.*

Proof. Theorem proves (i). Theorems and , with Proposition , prove (ii). Theorem and Theorem prove (iii). Theorems and prove (iv). Theorem proves (v). Theorem proves (vi). The constructions and their domains stop before the maps listed in (vii); no theorem of their existence or universal nonexistence is supplied.

The remaining sharp construction problem

A successor seeking the original nonlinear observable must provide:

1. a product domain and topology not satisfying all incompatible requirements (Q1)–(Q4);

2. a complete binary product family, including its generated counterterm outputs;

3. reconstruction and classical-presentation descent;

4. the associativity and comparison identities and ;

5. mixed time-ordered products based on actual functional support;

6. a compatible quantum gauge differential and the relevant multiple-insertion Ward identities;

7. a normalization catalogue and a specified physical state. One numerical finite part does not discharge any unproved entry in this list.

Conclusion

boxed

$$\begin{gathered}\text{sharp off-shell equality + microcausal polynomial admission} \quad \text{is incompatible at} \mathscr{B}_f,\\ \text{old Wick agreement + convergent sharp admission + hypocontinuity + continuous extraction} \quad \text{is incompatible at} \hbar^3,\\ \text{finite-width density preparation} \quad \Rightarrow \quad \text{admitted free insertion and products,}\\ \text{but its sharp transmission data are changed,}\\ \text{linear free-shell repair} \neq \text{nonlinear interacting gauge closure.}\end{gathered}$$

The three routes have been advanced as far as their proved definitions permit. None is called a solution merely because it changes notation, adds auxiliary variables, or assigns a finite number to the old triangle.

Declarations

Proof scope. The new proofs concern the preparation comparison, explicit convolution defect, density-ramp kernel admission, transmission law, boundary charge topology, auxiliary equivalence, and the stated algebraic interfaces. The quantitative sharp Wick incompatibility uses the explicit numerical VII-C/D input.

Preserved data. The original zero-Cauchy prescription, the restricted plateau gauge domain, and the old free product are not silently redefined. Each repair states the equality requirement or preparation datum that it changes.

Quantum scope. A free finite-width polynomial sector is constructed. No interacting quantum differential, quantum gravitational Ward identity, quantum retarded coefficient of the inherited characteristic graph, or ultraviolet completion is claimed.

State scope. The positive diagnostic is a scalar vacuum marginal on a fixed smooth classical metric. The Gaussian coefficient functional for the full gauge-fixed matrix is not asserted to be positive.

Dependency scope. The full supplied VII-A–D sources are cited with their actual scopes. Later manuscripts available only as supplied excerpts are comparison records, not unstated premises. No unpublished numbered assertion is fabricated.

Verification. No successful pdflatex execution, proof-assistant check, symbolic execution, numerical execution, independent referee review, or journal acceptance is asserted.

Book expansion notes (t433u)

None of the three repair routes removes the VII-B or VII-C obstruction [2, 3]. A finite-width free product is not the sharp product. XV-B stays closed; XVI-N is not discharged. Observation-boundary and Enter calculus records: [5], [25].

Standing ledger. Not spacetime emergence; not Einstein dynamics; not a Page curve, island, bounce, vacuum-energy selection, or de Sitter state as an Enter theorem; not a derived metric. O.1 stays NOT_CLOSED. XVI-N is not discharged. XV-B is not opened. No quantum coefficient is admitted. Host C is not created. Dom (T) remains undetermined; J2 remains unresolved. Problems 6, 7, and 9 are named, not solved. Gate 0 and ST4b stay NOT_SUPPLIED. Host-A stays NAMED_COMPARISON_ONLY $\{\mathrm{GI}, \mathrm{CB}\}$. The shredder is not Gibbons–Hawking. Cited sheets are source records, not premises. Conjectures stay conjectures. wired=false.

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