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Landmark VII-J Scheduled dependencies and certified physical causality in localized quantum simulation

Exact support, directional reachability, approximation, and continuation through a fixed-cone slicing event

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Abstract

We distinguish scheduled reachability, intervention-based quantum influence, and an independently supplied physical causal relation. For finite quantum networks, including their classical controllers, we prove that the complete timed dependency graph bounds the Heisenberg support of every jointly observed output. Absence of a scheduled path implies absence of signalling, including with an untouched entangled reference. A causal embedding of the dependency edges then gives an exact physical containment theorem for the encoded–computed–decoded channel.

The directional route is developed separately. A finite, phase-expanded periodic graph has a cycle-velocity polytope and an explicit finite-time remainder. Support-function comparison certifies convex velocity inclusion, but an asymptotic inclusion alone does not certify short-time communication. A spatially varying example gives an exact earliest-arrival law. For approximate simulation we prove the signalling bound with consistent diamond-norm constants and separate simulation accuracy from causal safety. A finite-volume Hamiltonian argument gives a reference-stable influence-tail certificate; a product-formula estimate supplies a corresponding simulation error.

For VII-F's three-to-six particle-link event, the ambient spinless cone remains fixed. Its uniformly spacelike,

ordered slices give a conservative causal gate speed $1/(1 + 2\sqrt{2}/3)$ in the slicing parameter. We certify physical inclusion across the event using persistent spatially marked registers, rather than abstract cell labels. An explicitly buffered, nontrivial two-qubit transport target is implemented exactly by causal routes and local gates, together with the separately localized event-code permutation. The target preserves arbitrary reference entanglement and has a unit signalling witness along an allowed causal route.

No spacetime metric, field equation, Lorentz invariance, or gravitational backreaction is derived. The finite causal carrier model is not a quantum-field discretization. The separate characteristic quantum-product programme is not advanced.

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par **Release and scope** par **Constructed:** exact network-support and physical-containment theorems; operational signalling criteria; localization and clock contracts; periodic directional bounds; robust finite-time margins; approximate signalling and simulation estimates; and a buffered causal realization through VII-F's event.

Two targets remain distinct. Target A concerns the complete decoded simulation channel. Target B additionally

embeds the actual elementary communication and gates in the supplied causal geometry.

Explicit model boundary. The cone application is a finite carrier-and-controller model on a prescribed metric cone. It does not derive matter propagation from Einstein dynamics or provide a continuum-field simulation.

No compilation, numerical execution, experimental implementation, or machine verification is asserted. par

The problem and the predecessor ledger

The central distinction is

$$\text{boxed scheduled dependency} \neq \text{actual quantum influence} \neq \text{geometric causal possibility}.$$

The aim is to prove relations among these objects after defining each one, not to identify them by terminology.

The finite-channel conventions are standard [1]. Operational causal notions require a stated intervention class; broader notions of influence need not agree in every probabilistic theory [2]. Circuit paths can overestimate the influence of a unitary transformation [3]. We use an intervention-based notion throughout.

What VII-E and VII-F actually provide

The supplied texts permit exact, manuscript-specific attribution.

toprule Supplied result Scope retained here

midrule VII-E, thm:locality, cor:local-readout Backward support for prescribed block channels and their readouts, with radius $(t - s)\Delta$, not radius $t - s$ unless $\Delta = 1$

VII-E, thm:retiming Equality of the total channel under legal reordering of fixed labelled channels; no clock identification

VII-E, prop:relay, prop:direction An attained relay speed and an anisotropic velocity diamond; these are schedule statements

VII-E, thm:pullback, thm:residual-obstruction Record compatibility and residual distinction, respectively; the

supplied labels are used rather than an unverified thm:record or thm:residual

VII-F, thm:certificate One particle-supported three-to-six slicing event on a fixed cone

VII-F, thm:implementation A forward-domain event-code circuit with retained history and identity action on internal matter; no causal embedding of its compiler gates

bottomrule

These statements are cited as manuscript results [15, 16]. The network and causal arguments used below are proved here.

VII-G's history-stability theorem is a compatible finite-channel comparison, not the physical event theorem [17]. VII-H's response distribution is neither a schedule nor an instrument [18]. The classical, domain, and numerical results of VII-A–D likewise do not construct the causal embedding studied here [11, 12, 13, 14].

Two meanings of causal fidelity

Definition (Observable simulation: Target A). Let $\Psi_{t,s}$ be a declared target channel. Let

$$\Phi_{t,s} = \mathscr{D}_t \circ \mathscr{C}_{t,s} \circ \mathscr{E}_s$$

be the implemented channel on the same model input and output spaces. Target A consists of:

1. a causal-support or signalling estimate for $\Phi_{t,s}$;

2. a separately proved accuracy estimate comparing $\Phi_{t,s}$ with $\Psi_{t,s}$. Intermediate compiler gates need not represent model events.

Definition (Causal elementary realization: Target B). Target B additionally assigns all implementation ports, communications, gates, preparations, measurements, and control records to events or regions in the supplied physical model. Each possible dependency has a certified causal route.

Target B is stronger. Endpoint causal support alone does not construct a local realization of an arbitrary channel. Our positive Target B constructions use a specified routing architecture and local gate primitives.

Finite channels and operational influence

Let the input factors be indexed by a finite set X:

$$\mathscr{H}_X = \mathrm{bigotimes}_{x \in X} \mathscr{H}_x.$$

For $S \subseteq X$, define

$$\mathscr{A}_X(S) = L\big(\mathrm{bigotimes}_{x \in S} \mathscr{H}_x\big) \otimes I_{X \setminus S}.$$

Output factors may have a different index set Y.

A channel is completely positive and trace preserving. Its adjoint is defined by

$$\mathrm{Tr}\,(B\Phi(Z)) = \mathrm{Tr}\,(\Phi^{*}(B)Z).$$

Adjoints are unital. We use

$$d_{\mathrm{tr}}(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1, \qquad d_{\diamond}(\Phi, \Psi) = \frac{1}{2}\|\Phi - \Psi\|_{\diamond}.$$

The diamond norm, without the factor 1/2, is denoted explicitly.

Admissible interventions

For input $A \subseteq X$, an intervention is any CPTP map

$$\mathscr{I}_A : L(\mathscr{H}_A) \to L(\mathscr{H}_A).$$

It is deterministic and nonselective. A locally measured outcome may be retained as an additional declared output, but conditional normalization of a selected branch is not an intervention in this definition.

For output $B \subseteq Y$, put

$$\Phi_B = \mathrm{Tr}_{Y \setminus B} \circ \Phi.$$

Definition (Reference-assisted signalling). Let R be an arbitrary finite untouched reference. Define

$$\mathrm{Sig}_{\Phi}(A \to B) = \tfrac{1}{2}\sup_{R,\rho,\mathcal{I}_{A'},\mathcal{J}_A} \| \ (\Phi_B \otimes \mathrm{id}_R)[(\mathcal{I}_A \otimes \mathrm{id}_{A^cR})(\rho) - (\mathcal{J}_A \otimes \mathrm{id}_{A^cR})(\rho)]\|_1.$$

The observer receives B and the untouched reference R. Thus $0 \le \mathrm{Sig}_{\Phi} \le 1$.

The reference may initially be entangled with the input. It is not acted upon by either intervention. This definition measures deterministic signalling, not postselected steering.

Theorem (Exact no-signalling criterion). *The following are equivalent:*

1. $\mathrm{Sig}_{\Phi}(A \to B) = 0$*;*

2. *there is a channel* $\Xi : L(\mathscr{H}_{A^c}) \to L(\mathscr{H}_B)$ *with*

$$\Phi_B = \Xi \circ \mathrm{Tr}_A;$$

3.

$$\Phi_B^*(L(\mathscr{H}_B)) \subseteq I_A \otimes L(\mathscr{H}_{A^c}).$$

The factorization implies equality with every untouched reference.

Proof. Factorization implies (i), since a local trace-preserving intervention leaves the A^cR reduction unchanged.

Suppose (i) holds. Compare the identity intervention with the replacement channel

$$\mathscr{T}_A(Z) = \mathrm{Tr}\,(Z) I_A/\dim \mathscr{H}_A.$$

With no reference, equality for every input state gives equality of the corresponding linear maps:

$$\Phi_B = \Phi_B(\mathscr{T}_A \otimes \mathrm{id}_{A^c}).$$

Define

$$\Xi(Z) = \Phi_B(I_A/\dim \mathscr{H}_A \otimes Z).$$

This is CPTP and gives (ii).

Taking adjoints proves (ii) implies (iii). Conversely, (iii) makes the output expectation depend only on the A^c

reduction. The same replacement construction gives (ii). Tensoring the factorization with id_R proves the final assertion.

For a joint output B, its *actual influence set* consists of input factors a for which $\mathrm{Sig}_{\Phi}(\{a\} \to B) > 0$. Equivalently, it is the union of tensor factors occurring in the minimal support of $\Phi_B^*(L(\mathscr{H}_B))$. The equivalence follows from Theorem and

$$\bigcap_{a \in A} \mathscr{A}_X(X \setminus \{a\}) = \mathscr{A}_X(X \setminus A).$$

Remark (Joint outputs matter). Separate no-signalling tests for B_1 and B_2 do not automatically certify the joint output B_1B_2. A classical bit can be encoded as the parity of two uniformly random shares: each share alone is independent of the bit, while their joint law reveals it. Our support certificates below apply to every declared jointly observed output.

The complete timed dependency network

Quantum and classical ports

A finite network is a directed acyclic circuit of channels with input and output ports, fixed initial ancillas, and declared discarded outputs. Its vertices include:

• quantum and classical register states;

• local gates and routed wire segments;

• ancilla preparations;

• destructive measurements and their outcome registers;

• resets, discards, guards, and classical feed-forward;

• every classical variable read to select a later operation.

For a conservative dependency graph, connect every input port of a gate to every output port of that gate. A tighter graph is allowed only with a proved joint-output support certificate for that gate. Identity wires have their ordinary

directed edges.

Classical registers use a distinguished basis. Controller maps first read the declared classical blocks and implement fixed branch maps. They are channels independent of an unknown input density operator. A canonical pinching extension allows the finite matrix-channel arguments to apply on a common ambient matrix space.

Contract (Complete control ledger). A controller may inspect only declared classical inputs. Every variable affecting the gate choice, routing, guard, availability time, or reset is included among the control ports.

A finite adaptive execution is padded to a fixed finite instruction envelope. Its dependency graph contains every possible branch and the control reads selecting that branch. No unrecorded state-functional scheduling is permitted. Ancillas are supplied independently of the unknown input; correlated ancilla preparation and transport must be declared.

A local measurement instrument is represented by its complete record-retaining channel. A fixed controlled continuation is then a channel on the record and the registers it reads. Padding unused branches with identities and retaining the controller gives the required finite envelope.

Scheduled ancestors

For a set of output ports B, let

$$\mathrm{Sch}^{-}(B) = \{\text{input factors having a directed dependency path to } B\}.$$

This graph depends on the prescribed implementation, not merely on the final channel.

Theorem (Scheduled support bounds actual influence). *For the channel $\mathscr{C}$ of a complete finite network,*

$$\mathscr{C}^{*}(\mathscr{A}_{Y}(B)) \subseteq \mathscr{A}_{X}(\mathrm{Sch}^{-}(B)).$$

Consequently,

$$\mathrm{Inf}^{-}_{\mathscr{C}}(B) \subseteq \mathrm{Sch}^{-}(B),$$

and every intervention on inputs disjoint from $\mathrm{Sch}^{-}(B)$ *has zero signalling to B, including with an untouched reference.*

Proof. Pull an output observable backwards through a topological order of the network. If it is the identity on all outputs of a gate, unitality of the adjoint makes it the identity on that gate's inputs. Otherwise, its possible support expands only to the declared predecessors of the observed gate outputs.

For the conservative all-to-all gate graph, these are exactly all gate inputs. Tensoring with untouched factors does not add other support. Fixed preparations introduce no unknown input. A discard pulls a scalar identity backwards; a reset is treated as its stated channel. Measurement records and controlled continuations are included as ordinary channel ports.

Repeating this process leaves support only on the scheduled ancestors. This proves . Theorem gives the remaining assertions.

This proof extends the prescribed-block mechanism of VII-E [15, thm:locality] by explicitly including the controller and all communication interfaces.

Example (A scheduled path with no surviving signal). Declare a two-register SWAP, followed by the same SWAP. Its dependency diagram contains cross-register paths, but the total channel is the identity. An intervention on the first input cannot change the second output. Thus scheduled reachability can strictly exceed actual influence.

Example (Hidden classical communication). Measure a computational bit at A, and prepare a distant output bit B equal to the result. The complete channel has $\mathrm{Sig}\,(A \to B) = 1$. Omitting the classical outcome wire from a quantum-only diagram gives an invalid support certificate.

Distances, arrivals, and three clocks

A fixed undirected register graph with positive edge weights has a shortest-path metric

$$d_{\mathrm{comp}}(u, v) = \inf_{\gamma\,:\,u \to v} \sum_{e \in \gamma} w_e.$$

Directed weights instead give an asymmetric extended distance. Weights representing routing cost are not automatically communication times or geometric lengths.

In a timed network, waiting and availability are represented explicitly. The earliest-arrival function is

$$T_{\mathrm{arr}}(u, v; s) = \inf \{t : \text{a time-respecting path from} (u, s) \text{to} (v, t)\}.$$

The value is $+\infty$ if there is no path. On a finite time-expanded DAG, dynamic programming in topological order computes the arrivals exactly.

The three clocks are

$$\tau_{\mathrm{depth}}, \qquad t_{\mathrm{hardware}}, \qquad t_{\mathrm{model}}.$$

A certificate assigns model times to ports and, separately, hardware start and finish times. Reordering independent gates can change depth or hardware time without changing the channel. It need not preserve the assigned model-time embedding.

Localization and the exact physical-containment theorem

Independent physical possibility

A physical comparison structure consists of a set of events $\mathscr{M}$ and a reflexive transitive relation preceq. It is specified independently of the computational schedule.

For point-localized ports, assign an event $p_v \in \mathscr{M}$. A scheduled edge $u \to v$ is physically certified when

$$p_u \operatorname{preceq} p_v.$$

For uncertain or extended localization regions $\mathscr{L}_u$, a sufficient robust condition is

$$p \operatorname{preceq} q \quad \text{for every} p \in \mathscr{L}_u,\ q \in \mathscr{L}_v.$$

This all-point requirement is conservative. A smaller allowance must specify the correlated endpoint choices and their routes.

Encoding and decoding are additional network components:

$$\Phi = \mathscr{D} \circ \mathscr{C} \circ \mathscr{E}.$$

Their spreads are included in the full network. There is no free nonlocal encoding before the support theorem begins.

Theorem (Exact encoded causal containment: Target A). *Suppose:*

1. *the entire encoded–computed–decoded experiment is a complete finite network;*

2. *every dependency edge has a physical endpoint certificate;*

3. *input and output localizations use the same independent causal relation. For a joint output B, let* $\mathrm{Phys}^{-}(B)$ *be the input factors whose assigned events can causally reach at least one output event in B. Then*

$$\text{boxed}\,\mathrm{Inf}^{-}_{\Phi}(B) \subseteq \mathrm{Sch}^{-}_{\mathrm{full}}(B) \subseteq \mathrm{Phys}^{-}(B).$$

Inputs outside $\mathrm{Phys}^{-}(B)$ *cannot signal to B, even with an untouched entangled reference.*

Proof. Theorem gives the first inclusion. Every scheduled path is a chain of certified causal endpoint relations. Transitivity gives a causal relation between its first and last events. Hence every scheduled ancestor belongs to the physical past. The signalling conclusion follows from Theorem .

The theorem is sufficient, not necessary. Extra scheduled dependencies can cancel in the complete channel.

Elementary realization rather than endpoint interpretation

Contract (Routing architecture). The implementation model permits:

1. exact carriage of a finite register along a prescribed future causal curve;

2. local unitary gates and complete instruments on registers meeting at the same station;

3. local preparation of a specified input-independent ancilla;

4. local storage, resets, and discards;

5. classical records carried along the same kind of causal communication routes. Gate durations are positive and explicitly assigned. These are ideal model primitives, not claims of laboratory realization or a quantum-field description of a point register.

Proposition (Gather–operate–distribute realization). *Consider a gate with input events* $p_1, \ldots, p_m$ *and output events* $q_1, \ldots, q_n$. *Suppose there are stations* r_-preceq r_+ *such that*

$$p_i \text{preceq}\, r_- \text{preceq}\, r_+ \text{preceq}\, q_j \quad \text{for all} i, j,$$

with specified routes, and the gate has a finite local implementation between r_- *and* r_+. *Then Contract gives a Target B realization of that gate.*

Proof. Carry the inputs to r_-, retain them through the local implementation, and carry the outputs from r_+ to their destinations. Include every ancilla and outcome register in the local implementation. Each wire and elementary gate has causal endpoints and a declared support. The endpoint channel is the intended gate, including with arbitrary references.

A causal relation between every input and output does not by itself imply the common-rendezvous hypothesis. We do not use the proposition as a universal localization theorem. QCA localizability and quantum causal graph dynamics provide stronger structural theorems under their own hypotheses [7, 8, 9]. They are comparison results, not unproved substitutes here.

Composing relations through an update

Proposition (Support composition through a transition). *Let*

$$\Phi = \Phi_+ \circ \mathscr{R} \circ \Phi_-,$$

where the three channels may act on differently indexed tensor products. Suppose their support predecessors are P_-, $P_{\mathscr{R}}$, *and* P_+, *respectively: each output algebra pulls back into the algebra of its declared predecessor set. Then*

$$\Phi^*(\mathscr{A}(B)) \subseteq \mathscr{A}(P_-(P_{\mathscr{R}}(P_+(B)))).$$

If all three predecessor relations have physical causal certificates in one common event space, the composite does also.

Proof. Compose the three adjoint inclusions. Compose the corresponding causal chains and use transitivity.

A change in register number is allowed if allocation and discard are channels with stated local supports. A newly prepared independent factor has no model-input predecessors. A state-dependent remapping must include its controller.

Directional paths and finite-time reachability

A velocity-set physical model

Let $K_{\mathrm{phys}}(x,t) \subset \mathbb{R}^d$ be a declared compact convex velocity set. Assume that the paths used in a certificate are absolutely continuous and satisfy

$$\dot{x}(t) \in K_{\mathrm{phys}}(x(t),t) \quad \text{almost everywhere.}$$

Define $p\mathrm{preceq}_K\, q$ when an admissible path joins the events with nondecreasing time. Concatenation makes this relation transitive.

For constant K, the reachable displacement set in time $T > 0$ is exactly TK: averages of allowed velocities lie in K, and a constant velocity realizes every point of TK.

Theorem (Local directional inclusion). *Suppose each scheduled communication has a specified continuous route with velocity in $K_{\mathrm{sched}}(x,t)$, and*

$$K_{\mathrm{sched}}(x,t) \subseteq K_{\mathrm{phys}}(x,t)$$

at every point visited by those routes. Include gate, encoding, decoding, and storage dependencies in the same path contract. Then the full schedule is physically contained, and Theorem applies.

Proof. Every scheduled route is an admissible physical path. Concatenated routes remain admissible piecewise

almost everywhere. The physical relation of each dependency edge is therefore certified.

The hypothesis is local along actual routes. A comparison of two average speeds is not a replacement for it.

Support functions and robust margins

For a compact convex set K, write

$$h_K(n) = \sup_{v \in K} n \cdot v.$$

Proposition (Directional certificate). *For compact convex $K, L \subset \mathbb{R}^d$,*

$$K \subseteq L \quad \Leftrightarrow \quad h_K(n) \le h_L(n) \quad \text{for every} n.$$

If $L = \{v : n_j \cdot v \le b_j,\ 1 \le j \le q\}$ is a polytope, it suffices to check $h_K(n_j) \le b_j$.

Proof. Inclusion gives the support inequalities. If $v \in K \setminus L$, let z be the Euclidean nearest point in L. The minimizing property gives $(v - z) \cdot (w - z) \le 0$ for every $w \in L$. Thus the direction $n = v - z$ strictly separates v from L, contradicting the support inequalities. The half-space claim is immediate.

Corollary (Uncertainty margin). *Suppose for all unit n,*

$$\begin{aligned} h_{K^{\text{true}}_{\text{sched}}}(n) &\le h_{\widehat{K}_{\text{sched}}}(n) + \delta_{\text{sched}}(n), \\ h_{K^{\text{true}}_{\text{phys}}}(n) &\ge h_{\widehat{K}_{\text{phys}}}(n) - \delta_{\text{phys}}(n). \end{aligned}$$

If

$$h_{\widehat{K}_{\text{phys}}}(n) - h_{\widehat{K}_{\text{sched}}}(n) \ge \delta_{\text{phys}}(n) + \delta_{\text{sched}}(n),$$

then true velocity inclusion holds. Strict inequality gives a positive certified margin.

Proof. Combine the inequalities and apply Proposition .

A phase-expanded periodic schedule

A periodic dependency model has a finite quotient graph Q. Its vertices include register type and schedule phase. Each edge e has displacement $\Delta_e \in \mathbb{R}^d$ and duration $T_e \ge T_{\min} > 0$. Its periodic lift permits translated copies of each

transition. Waiting at a port is allowed.

Assume Q is strongly connected. For a simple directed cycle c, set

$$\Delta(c) = \sum_{e \in c} \Delta_e, \qquad T(c) = \sum_{e \in c} T_e, \qquad v(c) = \frac{\Delta(c)}{T(c)}.$$

There are finitely many simple cycles. Define

$$K_{\mathrm{Sch}} = \operatorname{conv}(\{0\} \cup \{v(c) : c \text{simple cycle}\}).$$

Theorem (Cycle velocities with a finite-time offset). *Let $q = |V(Q)|$ and $D_{\max} = \max_e \|\Delta_e\|$. Every dependency path completed within elapsed time T satisfies*

$$\Delta_{\mathrm{path}} \in TK_{\mathrm{Sch}} + \overline{B}_{(q-1)D_{\max}}.$$

The set of limiting velocities along paths with elapsed times tending to infinity is exactly K_{Sch}. These are statements about potential scheduled reachability, not about the channel carried by every path.

Proof. Remove directed cycles from a finite walk until its remaining walk has no repeated vertex. Decompose each removed cycle into simple cycles. The remainder has at most $q - 1$ edges.

The cycle contribution is

$$\sum_c m_c \Delta(c) = \sum_c m_c T(c) v(c).$$

The sum of those cycle times is at most the elapsed time T. Including 0, the contribution belongs to TK_{Sch}. The remaining displacement has norm at most $(q - 1)D_{\max}$, proving .

The offset divided by T tends to zero, so every limiting velocity lies in K_{Sch}. Conversely, choose any finite convex combination of cycle velocities and zero. Repeat each chosen cycle sufficiently many times in the prescribed time proportions, connecting their quotient base vertices by fixed paths. Strong connectivity supplies the connectors. Their total duration and displacement are negligible relative to the repeated cycles. Additional waiting realizes the zero contribution. Integer repetition counts approximate arbitrary real proportions. This realizes every point of K_{Sch} .

For a quotient that is not strongly connected, one must respect the order of reachable strongly connected components. The convex hull of every cycle in the graph can overestimate attainability from a specified initial component.

The theorem is the timed, phase-explicit version of the periodic-graph mechanism discussed by Fritz [4]. It does not infer finite-time physical containment from an asymptotic polytope.

Example (An asymptotically stationary schedule can make a fast jump). Take two quotient states and edges with displacements L and $-L$, each of duration one, together with waiting. Every nontrivial cycle has zero displacement, so $K_{\mathrm{Sch}} = \{0\}$. The first edge nevertheless moves a dependency by L in one unit of time. For $L > c$ it violates a physical speed c. The finite offset in cannot be omitted.

The corrected diamond–disk comparison

For cardinal hops of length a, one per model interval τ, or a wait,

$$K_{\mathrm{Sch}} = \{v : |v_1| + |v_2| \le a/\tau\}.$$

For an isotropic physical speed $c = a/\tau$,

$$K_{\mathrm{Phys}} = \{v : \|v\|_2 \le c\}.$$

Then

$$K_{\mathrm{Sch}} \subseteq K_{\mathrm{Phys}}, \qquad K_{\mathrm{Phys}} \subseteq K_{\mathrm{Sch}}.$$

Indeed,

$$h_{K_{\mathrm{Sch}}}(n) = c\|n\|_\infty \le c\|n\|_2 = h_{K_{\mathrm{Phys}}}(n),$$

whereas the disk velocity $(c/\sqrt{2}, c/\sqrt{2})$ lies outside the diamond.

This corrects an orientation ambiguity in the proposed plan: the diamond is safe relative to the equal-speed disk; it does not cover every physically possible direction. VII-E's directional example concerns that failed reverse coverage [15, prop:direction].

Thus

$$\text{implementation influence} \subseteq \text{physical possibility}$$

is causal safety, while

$$\text{target dependency} \subseteq \text{available numerical stencil}$$

is an information-coverage requirement. Neither proves simulation accuracy.

A spatially varying causal benchmark

Choose $c_0 > 0$ and $0 \le t \le T$. In one spatial dimension let

$$K_{\mathrm{Phys}}(t) = \left[-\frac{c_0}{1+t}, \frac{c_0}{1+t}\right].$$

This is also the coordinate causal velocity set of the regular metric

$$\mathrm{d}s^2 = -\frac{c_0^2}{(1+t)^2}\mathrm{d}t^2 + \mathrm{d}x^2.$$

Proposition (Exact reachable set and earliest arrival). *Communication from* (s,x) *to* (t,y), $t \ge s$, *is possible exactly when*

$$|y-x| \le c_0 \log\frac{1+t}{1+s}.$$

Its earliest arrival is

$$T_{\mathrm{arr}}(x,y;s) = (1+s)\exp\left(|y-x|/c_0\right) - 1,$$

provided that value lies in the declared time window.

Proof. Integrating the velocity bound proves necessity. For sufficiency, put

$$x(r) = x + (y-x)\frac{\int_s^r c_0(1+u)^{-1}\,\mathrm{d}u}{\int_s^t c_0(1+u)^{-1}\,\mathrm{d}u}.$$

Its speed is bounded by $c_0/(1+r)$, and it reaches y. Solving the equality in gives the arrival formula.

A register route with this printed profile is certifiable even when a constant-speed route with the same endpoints is not. A controller may select among finitely many such routes only after its selecting record reaches the route's departure port. Preloading a known function $c(t)$ is not an instantaneous global feed-forward operation.

Taking an independently declared finite sequence of transport and local gates along these routes gives a nontrivial exact discrete target. Applying Theorem proves its causal safety; equality of the implemented gates with the declared target proves its accuracy. No wave-equation discretization is claimed.

Approximate observable simulation

The norm constant

Theorem (Reference-assisted signalling stability). *Let Φ, Ψ have the same input and output spaces. For a fixed output B, suppose*

$$\|\Phi_B - \Psi_B\|_\diamond \le \varepsilon_B.$$

Then

$$\operatorname{boxed}\operatorname{Sig}_\Phi(A \to B) \le \min\{1, \operatorname{Sig}_\Psi(A \to B) + \varepsilon_B\}.$$

In particular, a half-diamond error bound $d_\diamond(\Phi_B, \Psi_B) \le \varepsilon$ contributes 2ε, not ε, on the right.

Proof. For a fixed reference, input, and two interventions, let $\rho_{\mathcal{J}}$ and $\rho_{\mathcal{J}}$ be the resulting states. The two simulation-error terms obey

$$\|((\Phi_B - \Psi_B) \otimes \mathrm{id}_R)(\rho_{\mathcal{J}})\|_1 \le \varepsilon_B,$$

and the same holds for $\rho_{\mathcal{J}}$. The triangle inequality for the output difference therefore adds $2\varepsilon_B$ in trace norm and ε_B after the prefactor 1/2. Take the supremum and use $\operatorname{Sig}_\Phi \le 1$.

An expectation estimate for one state or one observable does not supply the diamond hypothesis. The local norm above still quantifies over every input and reference.

Composed error ledgers

Proposition (Channel telescope). *For channels Φ_j, Ψ_j between matching successive spaces, if*

$$\|\Phi_j - \Psi_j\|_\diamond \le \varepsilon_j,$$

then

$$\|\Phi_m \cdots \Phi_1 - \Psi_m \cdots \Psi_1\|_\diamond \le \sum_{j=1}^{m} \varepsilon_j.$$

This includes separate encoding, discretization, transition, compilation, decoding, and hardware stages when corresponding channel comparisons have actually been supplied.

Proof. Insert one difference at a time between unchanged channel prefixes and suffixes. Channels have diamond norm one, and the norm is submultiplicative. Apply the triangle inequality [1].

If a target has leakage at most $\varepsilon_{\mathrm{Phys}}$, the implemented leakage is at most

$$\min\left\{1, \varepsilon_{\mathrm{Phys}} + \varepsilon_{\mathrm{enc}} + \varepsilon_{\mathrm{disc}} + \varepsilon_{\mathrm{event}} + \varepsilon_{\mathrm{comp}} + \varepsilon_{\mathrm{dec}} + \varepsilon_{\mathrm{hw}}\right\}$$

in the full-diamond conventions above.

An uncertainty in a geometric arrival time is not automatically a channel-norm uncertainty. It can instead be handled by the robust geometric margins below, or by a separate dynamical continuity bound.

Proposition (Finite Hamiltonian perturbation bound). *For bounded self-adjoint time-dependent generators $H(t), \widetilde{H}(t)$, with the same initial unitary,*

$$\|U(t,s) - \widetilde{U}(t,s)\| \le \int_s^t \|H(r) - \widetilde{H}(r)\|\,\mathrm{d}r.$$

Consequently,

$$\|\mathrm{Ad}_U - \mathrm{Ad}_{\widetilde{U}}\|_\diamond \le 2\int_s^t \|H(r) - \widetilde{H}(r)\|\,\mathrm{d}r.$$

Proof. Differentiate $U(t,r)\widetilde{U}(r,s)$ and integrate. All surrounding propagators are unitary. For channels, expand

$UXU^{\dagger} - \widetilde{U}\widetilde{X}\widetilde{U}^{\dagger}$ into two terms, also after tensoring with an identity.

This is one valid way to turn a specified finite-dimensional geometry-dependent generator estimate into a channel error.

The Hamiltonian route: tails and a constructive error

Continuous-time local Hamiltonian evolution generally has tails, unlike an exact bounded-range circuit.

Lieb–Robinson theory supplies sharper volume-uniform bounds under suitable interaction assumptions [5]. We give a deliberately conservative finite-volume proof sufficient for this paper.

An explicit finite-volume tail

Let $G = (X, E)$ be a finite graph and

$$H(t) = \sum_{x \in X} h_x(t) + \sum_{e \in E} h_e(t),$$

where h_x acts at one site, h_e at the two endpoints of e, and all terms are bounded, self-adjoint, and piecewise continuous. Put

$$\Lambda(s,t) = \int_s^t \sum_{e \in E} \| h_e(r) \| \, \mathrm{d}r.$$

Let $d_G(A, B)$ be the minimum graph distance between the two nonempty disjoint regions.

Theorem (Reference-stable finite-volume influence tail). *For the unitary evolution channel $\Psi_{t,s}$, and $d = d_G(A, B) \geq 1$,*

$$\operatorname{boxed}\operatorname{Sig}_{\Psi_{t,s}}(A \to B) \leq \min\left\{1, \sum_{n=d}^{\infty} \frac{(2\Lambda(s,t))^n}{n!}\right\}.$$

For every $\mu > 0$, the tail is also bounded by

$$\exp\left(-\mu d + 2e^{\mu}\Lambda(s,t)\right).$$

The bounds are independent of the dimension of an untouched reference. They are not claimed uniform in volume.

Proof. Pass to the interaction picture for the onsite terms. Their propagator is a product of onsite unitaries and preserves every support. It conjugates each two-site interaction without changing its norm or support.

For an output observable $O_{BR} = O_{BR}^{\dagger}$ with $\|O_{BR}\| \le 1$, expand the interaction-picture Heisenberg evolution in its norm-convergent time-ordered nested-commutator series. An order-n term has norm at most

$$2^n \prod_{j=1}^{n} \|h_{e_j}(r_j)\|.$$

Summing the ordered integrals gives $(2\Lambda)^n/n!$.

A nonzero commutator with an edge term requires that edge to meet the current support. Each commutator can extend the graph distance from B by at most one. Thus every term of order $n < d$ acts as the identity on A. The same statement holds when the coefficients are arbitrary operators on R.

Let $O_{<d}$ be the partial sum of those terms, including the onsite conjugation. Its support excludes A, and the remainder has norm at most the tail in . The two intervention adjoints agree on $O_{<d}$ and are contractions in operator norm. Their difference on the remainder has norm at most twice the tail. Duality with the trace norm and the factor 1/2 in give .

For $n \ge d$, insert $1 \le e^{\mu(n-d)}$ and sum the exponential series.

One may choose an error-dependent influence radius by requiring the tail to be at most a prescribed $\varepsilon_{\mathrm{Phys}}$. This is an approximate dependency region of the named Hamiltonian model, not a strict Lorentzian cone.

Example (Strict causality fails for a generic local Hamiltonian). For two sites separated by a positive assigned geometric distance, take $H = JX_AX_B$, $J > 0$. Prepare B in $|0\rangle$, and use replacement interventions preparing A in $|+\rangle$ or $|-\rangle$. The two B outputs are

$$e^{-\mathrm{i}JtX}|0\rangle, \qquad e^{\mathrm{i}JtX}|0\rangle.$$

Their trace distance is $|\sin(2Jt)|$, nonzero for arbitrarily small positive t. A finite-dimensional interaction of this type therefore does not have an exact geometric propagation cutoff.

A product-formula simulation certificate

Let $H = \sum_{j=1}^{m} H_j$ be time independent and define

$$V_n(t) = \left(e^{-\mathrm{i}(t/n)H_m}\cdots e^{-\mathrm{i}(t/n)H_1}\right)^n, \qquad C_H = \sum_{j<k} \|[H_j, H_k]\|.$$

Proposition (Explicit first-order product-formula error). *For* $t \geq 0$,

$$\left\|e^{-\mathrm{i}tH} - V_n(t)\right\| \leq \frac{t^2}{2n} C_H, \qquad \left\|\mathrm{Ad}_{e^{-\mathrm{i}tH}} - \mathrm{Ad}_{V_n(t)}\right\|_\diamond \leq \frac{t^2}{n} C_H.$$

Proof. For two terms, a double Duhamel expansion compares their joint exponential with the ordered product; the error is bounded by $\delta^2\|[H_1, H_2]\|/2$ on an interval of length δ. Iterating over the finite sum gives the one-step bound $\delta^2 C_H/2$, since $\|[H_j, \sum_{k<j} H_k]\| \leq \sum_{k<j} \|[H_j, H_k]\|$. Telescoping n unitary steps yields the first estimate. The second follows from $\|\mathrm{Ad}_U - \mathrm{Ad}_V\|_\diamond \leq 2\|U - V\|$.

Combining Theorem , Proposition , and Theorem certifies the Hamiltonian route at Target A, including reference systems.

It does not automatically give Target B at the target Hamiltonian time: assigning every small nonlocal block gate an arbitrarily short geometric duration can violate the physical communication bound. A causal routing implementation needs a separate clock and latency certificate. More efficient local-Hamiltonian simulation algorithms use Lieb–Robinson decompositions [6]; no efficiency claim from that literature is needed for our elementary estimate.

The fixed cone and VII-F's slicing event

The independent causal geometry

Let $C_{\pi/2}$ be the spatial Euclidean cone of total opening $3\pi/2$, defined by gluing the boundary rays of that wedge, not by quotienting the whole plane by a finite rotation group. Let d_C be its intrinsic length distance. Define

$$\mathscr{M} = \mathbb{R}_T \mathrm{timesC}_{\pi/2}.$$

Away from the axis the metric is

$$-\,\mathrm{d}T^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2.$$

At the axis we use its metric-length completion.

A future causal curve is a locally absolutely continuous curve whose spatial metric speed is at most its nonnegative time derivative.

Lemma (Causality of the metric cone product). *For $p = (T, x)$, $q = (T', y)$,*

$$\mathrm{boxed}\,p\mathrm{preceq}\,q \quad \Leftrightarrow \quad T' - T \geq d_C(x, y).$$

This relation is reflexive and transitive, including at the axis.

Proof. Integration of the speed inequality proves necessity. The cone is a proper length space: bounded closed sets are compact. A minimizing sequence of spatial paths can be parametrized with uniformly bounded speed in a common compact ball; uniform convergence gives a length-minimizing path. Parametrize that path at speed at most one in the available time, inserting a wait if necessary. This proves sufficiency. The triangle inequality gives transitivity.

This is the physical comparison relation of the cone application. It is not inferred from a circuit graph. The spinless conical interpretation is classical [10]; no field dynamics is derived here.

The exact slicing family

In each of VII-F's three developed quadrants, write $x = \xi v_i + \eta v_{i+1}$, $\xi, \eta \geq 0$, and set

$$\rho(x) = \xi + \eta, \qquad H(x) = \frac{2}{3}(\xi + \eta), \qquad K(x) = \frac{3}{5}\min\{\xi, \eta\}.$$

The functions agree on identified rays. For $-1 \leq s \leq 1$, define

$$F_s(x) = s + \min\{2, \max\{H(x), s + K(x)\}\}, \qquad \Sigma_s = \{(F_s(x), x) : x\mathrm{in}\mathrm{C}_{\pi/2}\}.$$

VII-F proves the complete geometric certificate, including its affine holonomy, collar, and three-to-six particle

link [16, thm:certificate]. For self-containment of the causal argument, the required estimates are verified here.

Lemma (Uniform estimates through the event). *Let*

$$\lambda_0 = \frac{2\sqrt{2}}{3} < 1.$$

For every $s, s' \in [-1, 1]$, $s' \geq s$,

$$\begin{aligned} |F_s(x) - F_s(y)| &\leq \lambda_0 d_C(x, y), \\ s' - s &\leq F_{s'}(x) - F_s(x) \leq 2(s' - s). \end{aligned}$$

Every face is spacelike, with induced-metric lower bound $1 - \lambda_0^2 = 1/9$. *At* $s = 0$ *the limiting graph remains embedded and uniformly spacelike in the stratified Lipschitz sense.*

Proof. The nonconstant spatial gradients are

$$(2/3, 2/3), \qquad (3/5, 0), \qquad (0, 3/5)$$

up to rotations. Their norms are at most λ_0. Maximum and minimum preserve the common Lipschitz bound. Integrate it along spatial paths and take their length infimum.

For fixed x, the minimum–maximum expression after the leading s is nondecreasing and one-Lipschitz in s, proving the time estimates. The induced metric on an affine graph with gradient a is $I - aa^{\mathrm{T}}$, bounded below by $(1 - \lambda_0^2)I$. Projection of the graph to the cone is a homeomorphism. No vanished germ cell is counted as a positive-dimensional regular face at the event.

The spacetime axis and deficit remain fixed. The incidence changes at $s = 0$; it is not a degeneracy of the supplied causal relation.

Physical ports, not abstract cell labels

Choose finitely many persistent spatial marks

$$x_v \mathrm{in} \mathrm{C}_{\pi/2}.$$

Register v at slicing parameter s is localized at

$$p_v(s) = (F_s(x_v), x_v).$$

These marks are additional simulation data.

VII-F's abstract location transport is the identity on persistent labels, but some labels change their geometric realization. For example, an edge that formerly meets the particle need not meet it after the event. Identity of a cell name does not prove identity of a physical localization or a causal transport route. We therefore do not use that transport as a worldline certificate.

Theorem (Finite-time cone gate certificate). *For an input mark x at s and output mark y at $s' \geq s$, it is sufficient that*

$$\operatorname{boxed} s' - s \geq (1 + \lambda_0) d_C(x, y)$$

to certify $p_x(s) \operatorname{preceq} p_y(s')$. Consequently every complete network satisfying this inequality for all dependency edges, including its encoding, decoding, controls, and transition, obeys across $s = 0$.

Proof. By Lemma ,

$$F_{s'}(y) - F_s(x) \geq (s' - s) - \lambda_0 d_C(x, y).$$

Condition makes the right side at least $d_C(x, y)$. Apply Lemma and then Theorem . The estimates hold on both sides and at the event.

Corollary (A uniform conservative directional speed). *A spatial route parametrized by s, with metric speed at most*

$$v_* = \frac{1}{1 + \lambda_0} = \frac{1}{1 + 2\sqrt{2}/3},$$

lifts to a future causal curve $(F_s(x(s)), x(s))$. In each regular developed chart, the Euclidean ball $\{v : \|v\| \leq v_*\}$ is an admissible conservative velocity set in the slicing parameter.

Proof. For a short increment $\delta s > 0$,

$$F_{s+\delta s}(x(s+\delta s)) - F_s(x(s)) \geq \delta s - \lambda_0 d_C(x(s+\delta s), x(s)).$$

Taking the almost-everywhere metric derivative gives

$$\frac{\mathrm{d}T}{\mathrm{d}s} \geq 1 - \lambda_0 |\dot{x}|.$$

The speed restriction makes this at least $|\dot{x}|$. Thus the lifted curve is causal.

This is an inner causal bound, not the entire physical velocity set. No equality with a schedule cone is asserted.

Localization and timing uncertainty

Proposition (Robust cone margin). *A nominal dependency has slicing-time gap h, nominal endpoint distance at most D, spatial localization radii r_-, r_+, slicing-time errors at most ε_s at each endpoint, and additional absolute spacetime-time errors at most ε_T at each endpoint.*

It remains causally admissible if

$$h - (1+\lambda_0)D \geq (1+\lambda_0)(r_- + r_+) + 2\varepsilon_s + 2\varepsilon_T.$$

Strict inequality is a positive robustness margin.

Proof. The actual spatial distance is at most $D + r_- + r_+$, and the actual slicing gap is at least $h - 2\varepsilon_s$. The time-difference estimate of Theorem then loses at most another $2\varepsilon_T$. Rearrangement gives .

The event guard of VII-F is an exact stored-clock test. This margin concerns physical port placement and availability. It is not a replacement of that guard by a rounded edge report.

The event transition: exact macro fidelity and buffered locality

What must be added to the event-code theorem

Fix any one finite forward-domain event circuit U_{code} from VII-F. Its input, history, work, and code registers are finite; work is returned to zero on the forward domain, and history is retained [16, thm:implementation].

Two additional choices are essential:

1. all code and location-descriptor bits are placed at a declared controller station;

2. a model-time interval is assigned to their actual finite computation. A location descriptor stored at the station is not a remote quantum register. No instantaneous acquisition of unknown remote location data is implied.

For Target A, one may interpret U_{code} as a macro event on local code data with identity matter channel. For Target B, positive-duration gates require a time buffer.

The code's germ timestamp $s = 0$ is data describing the geometric event. It is not the time at which every computed code bit becomes physically available. A computation announced after the event is not an instantaneous global geometric update.

A nontrivial finite target through the event

Let o be the cone apex and choose the persistent ray mark

$$e = (r_*, 0), \qquad r_* = \frac{1}{64}.$$

Set

$$h_* = \frac{1}{4}.$$

Place one input qubit at each of $p_o(-h_*)$ and $p_e(-h_*)$. The independent target is:

1. update the local event code by its specified U_{code};

2. exchange the two quantum states at the output ports $p_o(h_*)$, $p_e(h_*)$;

3. retain code history and return clean work. Thus its matter channel is $\mathrm{Ad}_{\mathrm{SWAP}}$, not the identity. This is a declared discrete carrier target, not an Einstein or scalar-field evolution.

The encoding is the identity on each input qubit, with local clean code/work initialization. The decoding reads the returned target factors and the declared event record; it has no nonlocal preprocessing.

Lemma (Endpoint times and strict causal slack). *The four matter-port times are*

$$\begin{array}{ccc} c|cc & o & e \\ \text{hline}\, s = -h_* & -h_* & -h_* + 2r_*/3 \\ s = h_* & 2h_* & 2h_*. \end{array}$$

Both cross-port dependencies are causal. Their conservative slicing-time margin obeys

$$2h_* - (1 + \lambda_0)r_* > \frac{15}{32}.$$

Proof. On a ray, $K = 0$ and $H = 2r_*/3$. At negative s, the H branch is active; at positive $h_* > 2r_*/3$, the s branch is active. The apex time is s before and $2s$ after the event. Since $d_C(o, e) = r_*$, the causal claims follow from Theorem . Finally, $\lambda_0 < 1$ gives

$$\frac{1}{2} - \frac{1 + \lambda_0}{64} > \frac{1}{2} - \frac{1}{32} = \frac{15}{32}.$$

An elementary causal realization

Use the apex as the controller and rendezvous station. The model admits local finite quantum registers on its timelike axis under Contract ; no quantum-field boundary condition at the singular axis is asserted.

The qubit starting at e travels radially inward at unit spacetime speed. It can arrive at the apex at

$$-h_* + \frac{2r_*}{3} + r_* = -h_* + \frac{5r_*}{3} < 0.$$

The other qubit is already there. Both wait until $T = 0$.

Let L be the finite number of elementary gates in the event-code circuit plus three CNOT gates implementing SWAP. Execute them at the apex, each with assigned spacetime duration

$$\delta_T = \frac{h_*}{2(L + 1)}.$$

All gates finish before $T = h_*/2$. Route the appropriate output qubit to e and retain the other at the apex. Since

$$h_*/2 + r_* < 2h_*,$$

both output ports are reached in time, with waiting as needed. The code remains at the apex.

Theorem (Exact causal and accurate event benchmark). *For this declared target and architecture:*

1. *every route and every elementary gate is causally admissible in the independently supplied cone;*

2. *the implemented endpoint channel equals*

$$\mathrm{Ad}_{U_{\mathrm{code}}} \otimes \mathrm{Ad}_{\mathrm{SWAP}},$$

with histories retained and work returned on the canonical forward domain;

3. *accuracy is exact, including with every untouched entangled reference;*

4. *the two cross-port matter signalling strengths are one;*

5. *the model-time computation is compatible with the geometric event at* $s = 0$*, without assigning a geometric meaning to intermediate compiler code states.*

Proof. The incoming and outgoing radial trajectories obey $|x| \le T$. Storage is along the fixed timelike axis or the fixed ray station. Every elementary gate is local at the apex with positive duration. Thus Target B holds.

Transport carries the same tensor factors without measurement. At the rendezvous, the three CNOT gates give the SWAP exactly, while the independent code gates give U_{code}. The two operations act on distinct factors, so the endpoint channel is their tensor product. Unitary identities remain identities with every reference.

For signalling from the apex input to the ray output, compare replacement interventions preparing $|0\rangle$ and $|1\rangle$. SWAP gives orthogonal ray outputs, with trace distance one. The reverse direction is identical. These signals follow causal routes, rather than exceeding physical possibility.

The code describes the event germ at timestamp zero and becomes available after its finite computation. The supplied slicing passes continuously through zero during the bracket. Its metric and holonomy do not depend on the compiler states. The decoder statement concerns the canonical completed code, exactly as in VII-F.

A hardware implementation may give each local gate a hardware duration $\delta_{\mathrm{hw}} > 0$. The certificate records

$$t_{\mathrm{hardware},j} = j\delta_{\mathrm{hw}}, \qquad T_j = j\delta_T,$$

during the local stage. Along the positive-time apex worldline, $s_j = T_j/2$. Logical gate depth, hardware duration, and the model's slicing parameter are therefore explicitly different quantities.

The buffer is not a change in the cone's event. It is an added implementation latency around that event. An exact event code at model timestamp zero does not justify exposing its arbitrary compiler intermediate states as geometric configurations.

Pre-event, transition, and post-event composition

Corollary (Certified continuation through the particle-link event). *Place any finite pre-event and post-event networks on the slices , with complete control ledgers and the edge certificates . Insert either:*

1. *the local identity-on-matter macro event with independently certified code availability; or*

2. *the buffered benchmark of Theorem . The complete decoded network satisfies . If its three stage channels have full-diamond errors $\varepsilon_-, \varepsilon_{\mathscr{R}}, \varepsilon_+$ against a declared target, the total error is at most their sum.*

Proof. Use Proposition , Theorem , and Proposition .

This certificate does not require the graph of cell incidence to remain fixed. It requires the registers, routes, and causal port relation to remain specified. A new graph edge cannot be used as an instantaneous communication link merely because a geometric cell has been born.

What an instantaneous remapping would get wrong

Proposition (Zero-duration remote SWAP is excluded). *Two distinct points on any one slice Σ_s are spacelike separated. A same-slice remote SWAP has unit signalling between them and is not a causally faithful transition for the cone relation.*

Proof. The slice estimate gives

$$|F_s(x) - F_s(y)| \le \lambda_0 d_C(x,y) < d_C(x,y)$$

for distinct points. Lemma therefore excludes either causal direction. The replacement-state SWAP test gives unit signalling, as in Theorem .

Thus pre-event and post-event spacelikeness, event reversibility, and an unchanged deficit do not by themselves certify transition locality.

Benchmarks and failure modes

Static lattice: schedule, influence, and physical cone

A cardinal-hop schedule has the velocity diamond of . Replacing its two-site operations by identities leaves the same conservative support envelopes but removes cross-site influence. Replacing a permitted operation by SWAP can realize unit influence on that edge.

Assigning every cardinal dependency duration $\tau \geq a/c$ gives causal endpoint inclusion in a flat speed-c geometry. Changing only hardware parallelism does not change that requirement. This finite example separates all three structures.

For a nontrivial accuracy test, prescribe SWAP as the target. The identity implementation is safe, but

$$d_{\diamond}(\mathrm{id}, \mathrm{Ad}_{\mathrm{SWAP}}) = 1$$

on two qubits: the input $|01\rangle$ gives orthogonal outputs $|01\rangle$ and $|10\rangle$, while the half-diamond distance between channels is at most one.

Nonlocal encoding

Let two model sites be spacelike separated during a short comparison interval. An encoding that swaps their factors, followed by an identity computation and local decoding, gives a decoded SWAP. The computation alone has no communication, but the encoded–computed–decoded channel does. This violates the localization contract and has unit unwanted signalling.

Global feed-forward

A distant controlled gate can signal through its classical control even if its quantum data wires are local. It must receive the control record after a certified causal route. Declaring the controller ``external'' does not remove the dependency if its value depends on model inputs.

Postselection

Measurement on one half of an entangled state can change the conditional remote state. Ignoring the outcome preserves the remote reduction. A causal claim about an outcome-conditioned output must include the success flag and its communication. The deterministic intervention theorem does not assert absence of steering after conditioning.

Depth is not elapsed model time

A long-range SWAP can have constant abstract gate depth. If its port separation is D, its geometric communication latency cannot be inferred from that depth. Conversely, many local compiler gates at one station can have large depth without expanding the spatial influence region.

Geometric consistency remains necessary

VII-F's seam-compatible family is essential: its unequal-slope vacuum maximum fails to descend through the cone seam [16, lem:transplant]. A causal certificate cannot repair a multivalued geometric localization.

Nor may one replace the fixed full affine seam word by a time-shifted word with the same rotation. The marked fixed axis and the actual spacetime are part of the physical model. A different deficit or time translation is a different comparison problem, not a scheduling convention.

A reusable mathematical certificate

Definition (Causality and accuracy certificate). A certificate consists of the following finite data and proved bounds.

1. Physical structure: event space, causal relation, time window, and continuation through any singular or incidence event.

2. Target dynamics: a named channel Ψ, independent of the certificate's causal-safety conclusion.

3. Localization: model, encoding, ancilla, routing, controller, and decoding ports, with any uncertainty regions.

4. Complete dependency network: jointly valid support envelopes, including classical branches, resets, and discarded systems.

5. Clocks: logical depth, hardware availability, model port times, and their declared calibration.

6. Finite-time causal proof: edge routes or endpoint certificates, with margins. An asymptotic velocity polytope is recorded separately.

7. Event transition: its channel, locality, record-availability time, and allocation or remapping of every affected factor.

8. Accuracy: exact equality or a justified channel-norm bound against Ψ, including all stated references.

9. Scope: output regions, intervention class, parameter domain, and unsupported physical identifications.

Theorem (Soundness of the certificate). *Suppose the finite data satisfy Definition , with exact edge containment and target leakage* $\varepsilon_{\mathrm{Phys}}(A, B)$*. Then:*

1. *the complete implementation has the scheduled support of Theorem ;*

2. *exact causal edge containment gives ;*

3. *if the reduced full-diamond simulation error is at most* ε_B*, then*

$$\mathrm{Sig}_{\Phi}(A \to B) \leq \min\{1, \varepsilon_{\mathrm{Phys}}(A, B) + \varepsilon_B\};$$

4. *certified pre-event, transition, and post-event stages compose without a new instantaneous remapping assumption;*

5. *a routing-and-local-gate realization satisfying Contract also meets Target B.*

Proof. The assertions are Theorems , , and , Proposition , and the elementary routing construction. Each ingredient is proved for complete channels, so entangled input references are retained.

toprule Route Exact achievement and limit

midrule General macro-step Target A from adjoint support and a complete decoded-channel certificate; no automatic local implementation of an arbitrary macro map

Elementary routed circuit Target B when causal transport, rendezvous, local gates, and positive timings are printed

Homogeneous directional Support-function inclusion and a periodic cycle polytope; finite-time offset remains

Varying causal geometry Local admissible paths and an explicitly solved time-dependent velocity benchmark

Hamiltonian Reference-stable finite-volume tails plus a product-formula accuracy bound; no strict cone

VII-F event Uniform causal slot bound, robust margins, transition composition, and a nontrivial exact buffered carrier example

bottomrule

The certificate is an analytical mathematical object. A script checking a finite example can corroborate its arithmetic, but a pass flag is not a proof of its universal hypotheses.

Release, boundaries, and successor obligations

Theorem (VII-J release). *Within the finite models and contracts stated here:*

1. *actual reference-assisted influence is bounded by the complete scheduled ancestor set;*

2. *an independently certified physical embedding yields exact directional causal containment;*

3. *approximate decoded simulation obeys the norm-consistent signalling bound ;*

4. *finite-time periodic propagation, varying-velocity communication, and Hamiltonian-tail routes have the stated independent certificates;*

5. *the fixed-cone slicing event preserves the supplied causal relation, and causally certified transition channels compose through it;*

6. *the printed buffered event benchmark is both causally realizable and exactly accurate for its declared nontrivial quantum transport target. No identification of scheduled reachability with the whole geometric causal past is made.*

Proof. Combine Theorems , , , , , , and , with Propositions , , and .

The result closes the requested causal-comparison problem for explicitly declared finite architectures. It does not close every possible simulation problem on the cone.

A continuum-field successor would additionally need a specified field algebra and state, local encoding of its admitted observables, a target propagator or evolution, a discretization error, and a physical detector model. The finite transport target here supplies none of those by analogy.

Likewise, this paper does not construct the separate characteristic quantum coefficient left open in the supplied XIV–XV material. Its causal network proofs do not define a missing distributional product, choose an extension, or supply a quantum Ward identity. No O.1 or quantum admission status is changed.

The distinction from gravitational dynamics is unchanged:

certified matter communication on a fixed cone ≠ matter-driven evolution of the cone.

The cone metric, deficit, and worldline are inputs. No Lorentz invariance or spacetime emergence is derived from the schedule. No black-hole, recovery, Page-curve, or holographic claim is made.

Conclusion

boxed

$$\begin{gathered}\text{complete localized network + explicit clocks + independent causal edge certificates}\\ \Rightarrow \mathrm{Inf}^{-}_{\Phi}(B) \subseteq \mathrm{Sch}^{-}_{\mathrm{full}}(B) \subseteq \mathrm{Phys}^{-}(B),\\ \|\Phi_B - \Psi_B\|_{\diamond} \le \varepsilon_B \Rightarrow \mathrm{Sig}_{\Phi}(A \to B) \le \mathrm{Sig}_{\Psi}(A \to B) + \varepsilon_B,\\ s' - s \ge \left(1 + \tfrac{2\sqrt{2}}{3}\right) d_C(x,y) \Rightarrow (F_s(x), x)\,\mathrm{preceq}\,(F_{s'}(y), y).\end{gathered}$$

The first line is causal safety. A separate comparison with Ψ is simulation accuracy. The last line is the explicit

bridge to VII-F's event, not an assertion that its cell adjacency graph is a physical light cone.

Declarations

Proof scope. The new results concern finite channels and complete control networks, timed causal embeddings, directional and finite-time bounds, norm-stable signalling, an elementary Hamiltonian-tail estimate, and a buffered fixed-cone carrier realization.

Physical scope. The causal relation and the ideal transport architecture are declared model inputs. Their consistency is proved, but a laboratory realization or quantum-field derivation of those primitives is not asserted.

Event scope. VII-F supplies the geometric three-to-six certificate and finite event-code permutation. This paper adds physical localization, positive-duration implementation timing, causal routing, and transition containment. The stored event timestamp is not an instantaneous broadcast.

Reference scope. References to Cheung's works identify the supplied manuscripts and their stated results. They are not claims of journal publication, independent verification, or transfer between inequivalent models.

Verification. No successful pdflatex execution, proof-assistant check, solver run, independently executed numerical certificate, referee review, or physical implementation is asserted.

appendix

A finite implementation ledger for the cone benchmark

Fix one VII-F code instance with report precision b, clock precision d, permission width r, and n original location descriptors. Its displayed bound is

$$M_{\mathrm{code}} = 96b + 4d + 4r + 338 + 12n,$$

including retained history and clean work. Let G_{code} be any finite elementary gate count furnished by that instance.

The additional transport benchmark uses:

• two quantum matter registers;

• three CNOT gates for SWAP;

• causal carriage and storage of those two tensor factors, without measurement;

• no auxiliary matter register. Thus $L = G_{\mathrm{code}} + 3$ is a finite sufficient count for the local apex stage.

All code wires are at the apex station. The two matter wires are gathered there before $T = 0$. With

$$\delta_T = \frac{1}{8(L+1)},$$

the local stage ends before $T = 1/8$, since $h_* = 1/4$. The output deadline is $T = 1/2$. The radial route of length $1/64$ then leaves strict positive slack.

The event code is decoded only at the completed forward boundary, with history omitted from the geometric observation but retained physically. No name descriptor is used to change the matter routing; the two-ray transport script is fixed in advance. If a later script reads code-dependent routing, the corresponding record wires and their arrival times must be added to the network.

Why co-located gates do not settle arbitrary macro locality

Suppose a macro channel satisfies a physical endpoint no-signalling condition. This says what its outputs can depend upon, not how to generate them. A realization may require resource states, communication, or a decomposition whose ordinary circuit paths overestimate its actual influence.

The gather–operate–distribute proof avoids this gap by imposing a common station and explicit routes. The QCA localizability theorem [8] and quantum causal graph dynamics [9] address broader settings under additional hypotheses. Their graph notion of causality does not automatically supply the geometric port timings of this paper.

Consequently:

$$\text{macro no-signalling} \not\Rightarrow \text{every proposed compiler is causally embedded.}$$

Our Target B claim is for the printed compiler architecture and routes, not every representation of the same endpoint channel.

Book expansion notes (t433u)

Scheduled reachability is not a light cone. XV-B stays closed. The fixed-cone slicing of VII-F [16] is an input, not a derived causal structure. Observation-boundary and Enter calculus records: [15], [24].

Standing ledger. Not spacetime emergence; not Einstein dynamics; not a Page curve, island, bounce, vacuum-energy selection, or de Sitter state as an Enter theorem; not a derived metric. O.1 stays NOT_CLOSED. XVI-N is not discharged. XV-B is not opened. No quantum coefficient is admitted. Host C is not created. Dom (T) remains undetermined; J2 remains unresolved. Problems 6, 7, and 9 are named, not solved. Gate 0 and ST4b stay NOT_SUPPLIED. Host-A stays NAMED_COMPARISON_ONLY $\{\mathrm{GI}, \mathrm{CB}\}$. The shredder is not Gibbons–Hawking. Cited sheets are source records, not premises. Conjectures stay conjectures. wired=false.

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