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VII-J-ST: Scheduled dependencies and certified causality on a prescribed conical spacetime

A local Grade S Enter adaptation with complete clock instruments, directional certificates, and buffered regular-part transport

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Abstract

We give a self-contained local Grade S adaptation of VII-J's finite-network causality programme to I-B's prescribed 3 + 1-dimensional conical product. The metric, deficit $\pi/2$, causal relation, slicing family, preparations, and communication permissions are inputs. Complete timed dependency graphs, including classical controllers, measurements, records, encoders, and decoders, bound the Heisenberg support of every jointly observed output. A causal embedding then bounds reference-assisted intervention influence by the prescribed spacetime causal relation.

The operational addition is an explicit regular-part realization through the parameter of VII-F's transverse three-to-six incidence. Unlike the source apex-register benchmark, no quantum register, gate, or communication path meets the singular worldsheet. A destructive two-qubit clock controls finite event metadata; a buffered gather–operate–distribute protocol implements a nontrivial two-qubit exchange, preserves arbitrary retained reference correlations, and has a unit signalling witness on an allowed route. The complete recorded channel, positive durations, blank registers, controller storage, copied records, and finite instruction reserves are specified.

Directional certificates retain both cycle-velocity polytopes and finite-time offsets. A finite potential inequality

exposes transient directional costs that an asymptotic velocity inclusion can miss. Approximate simulation is treated separately: signalling changes by at most twice the half-diamond error, while a finite-volume Hamiltonian tail is reference-stable but not an exact Lorentzian cutoff. These assembled certificates may provide a useful reproducible benchmark for distinguishing logical simulation accuracy, operational influence, and elementary causal realizability. The novelty claimed is the supported assembly and regular-part correction, not the general principles of channel locality, periodic-graph velocities, or destructive measurement. No continuum-field simulation, spacetime emergence, Einstein dynamics, or inherited quantum response coefficient is constructed.

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par **Release boundary** par This manuscript proves a finite operational certificate on an installed classical geometry. The geometric event is a prescribed incidence change. It is not caused, detected at the singularity, or reconstructed by the finite clock.

The source VII-J benchmark permits a register on the apex. That permission is incompatible with the regular-part laboratory used here. We therefore construct a different, explicitly regular benchmark rather than silently

transferring the apex construction. par

Scope, Enter philosophy, and the two targets

VII-J separates scheduled dependencies, intervention influence, and an independently supplied causal structure [1]. I-B equips a prescribed conical product with finite names, accessible algebras, clock calibration, and communication permissions [2]. We retain those distinctions and prove the interfaces needed for their local combination.

The Enter discipline

Cheung's Enter framework treats destructive readout as an input–output and access discipline: measured quantum names are consumed, classical outcomes are returned, and specified unmeasured factors survive [3, 4]. This does not identify a classical record with the return of an unknown quantum input. Nor does a channel equality identify a thermal implementation, a geometric interpretation, or a gravitational dynamical law [5].

We adopt an explicit finite Born instrument. Its channel identity is proved below. We do not rederive the structural measurement characterization of Manuscript I from an abbreviated hypothesis list, and do not extend that characterization to fields.

The operational motivation can be summarized as four questions: what is supplied, what may be accessed, what is returned, and what additional construction supports the proposed interpretation? The present answer supplies a classical background and a finite network; it returns classical records and surviving finite quantum payloads.

Definition (Local Grade S). A local Grade S certificate consists of:

1. prescribed classical geometry and a supported regular operational presentation;

2. explicit finite channels and complete control interfaces;

3. a finite destructive-clock instrument with its calibration;

4. proved support, influence, and causal-embedding statements;

5. a nonempty finite transport realization with a resource ledger. Derivation of the geometry from operational records, called Grade E here, is not included.

Definition (Targets A and B). Let Ψ be a declared finite target channel and

$$\Phi = \mathscr{D} \circ \mathscr{C} \circ \mathscr{E}$$

an implementation on the same comparison interfaces.

Target A requires a support or signalling statement for Φ, together with a separate accuracy comparison to Ψ.

Target B additionally realizes every preparation, gate, communication, clock operation, controller read, record, reset, and decoder by the admitted local and causal primitives on the supplied spacetime.

Endpoint channel correctness does not by itself construct Target B. A physical embedding of dependency endpoints is a sufficient support certificate; an actual elementary realization also needs the declared implementation primitives.

Provenance. VII-J is the source causality manuscript; VII-F supplies the prescribed slicing family; I-B supplies the spacetime-interface pattern [1, 6, 2]. Other sheets are documentary comparisons. Their citation does not import unprinted proof hypotheses. Every positive mathematical claim used in this release has a proof below.

Prescribed spacetime, support, and the incidence input

Fix a length unit $\ell > 0$, a speed $c > 0$, and a time origin T_*. Write

$$\tau = \frac{c(T - T_*)}{\ell}, \qquad \mu = \frac{\pi}{2}, \qquad \alpha = \frac{3}{4}.$$

The regular background is

$$M = \mathbb{R}_T \times C^\circ_\mu \times \mathbb{R}_z, \qquad g = -c^2 \mathrm{d}T^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2,$$

where $r > 0$, φ has period 2π, and the stipulated completion has singular locus

$$\mathscr{W} = \mathbb{R}_T \times \{o\} \times \mathbb{R}_z.$$

The defect is a worldsheet. Only its fixed-z sections are worldlines. No smooth chart, pointwise Einstein tensor, or field boundary condition is asserted on $\mathscr{W}$.

On a lifted regular angular interval,

$$(T, r, \varphi, z)\mathrm{longmapsto}\,(c(T - T_*), r\cos(\alpha\varphi), r\sin(\alpha\varphi), z)$$

is a local Minkowski development. Static observers have $\mathrm{d}\tau_{\mathrm{proper}} = \mathrm{d}T$. The metric lapse in T is one. The spinless seam gluing fixes T, z and rotates the transverse developed plane by $\pi/2$ in the chosen gluing convention. These are the prescribed I-B data [2].

Proposition (Non-global hyperbolicity). *The regular spacetime M is not globally hyperbolic.*

Proof. Fix $r_0 > 0$, an angle, and $z = 0$. Take two events at that spatial point with times $T_* \pm 2r_0/c$. Events at time T_* and radii $r_n \mathrm{downarrow}\, 0$, at the same angle and axial coordinate, lie in their causal diamond by radial causal paths. They have no convergent subsequence in M. The diamond is therefore noncompact.

No global Green or Cauchy theorem is inferred. All operational paths below lie in a relatively compact regular developing tube.

The prescribed transverse event

Use a cut wedge of angle $3\pi/2$, divided into three right-angle sectors. In each sector let the dimensionless coordinates be $\xi, \eta \geq 0$, and put

$$\rho = \xi + \eta, \qquad h = \frac{2}{3}(\xi + \eta), \qquad k = \frac{3}{5}\min\{\xi, \eta\}, \qquad F_s = s + \min\{2, \max\{h, s + k\}\}, \quad -1 \leq s \leq 1.$$

The functions agree on identified sector rays. The prescribed product markings are

$$\widehat{\Sigma}_s : \quad \tau = F_s(x), \qquad z = \ell\zeta,$$

with no axial dependence in F_s. The formula is VII-F's transverse family, explicitly lifted rather than identified with a three-dimensional cell count [6].

Lemma (Estimates used by the causal certificate). *Set*

$$\lambda = \frac{2\sqrt{2}}{3} < 1.$$

On every regular developed region, F_s is spatially λ-Lipschitz, including the axial variable. For $s' > s$,

$$s' - s \le F_{s'}(x) - F_s(x) \le 2(s' - s).$$

The graphs are embedded, and their regular affine pieces are spacelike with induced dimensionless metric bounded below by $I/9$.

The prescribed transverse cellulation has one incidence-event parameter, $s = 0$, where its singular transverse link changes from three incident faces to six.

Proof. The active gradients are

$$(2/3, 2/3, 0), \quad (3/5, 0, 0), \quad (0, 3/5, 0), \quad 0,$$

up to sector rotations. Minima and maxima preserve their common Lipschitz bound. The graph metric is $I - aa^{\mathrm{T}}$; its least eigenvalue is at least $1 - \lambda^2 = 1/9$. Graph projection gives embeddedness. For fixed x, the term after the initial s in is nondecreasing and 1-Lipschitz in s, proving .

For the incidence, use

$$h - k \ge \frac{11}{30}\rho.$$

When $s \le 0$, the h-branch dominates and one face in each sector meets the apex. When $s > 0$, in the half-sector $\xi \ge \eta$, the new boundary is $10\xi + \eta = 15s$; its symmetric counterpart is $\xi + 10\eta = 15s$. Their vertices are

$$B_i = \frac{3s}{2} e_i, \qquad C_i = \frac{15s}{11}(e_i + e_{i+1}).$$

They define two distinct new apex triangles in each sector. Their outermost radial coordinate is $30s/11 < 3$, so they do not meet the persistent crest $\rho = 3$. Within each open epoch these boundaries have fixed combinatorics, and all new coordinates vanish only at zero. Thus the normal transverse link changes exactly once from three to six.

The source's full lengths, areas, affine chart words, primitive winding, and peripheral matching remain documentary geometric data [6]. They are not needed to infer the finite-channel support theorem. In the product, the event locus is the line $\{\tau = 0, x = o\} \subset \mathscr{W}$; it is not an isolated point event of the 3 + 1-dimensional background.

Definition (Supported operational presentation). A geometric atlas may describe a finite transverse patch and a finite axial slab in the conical completion, including its stipulated singular stratum.

The operational support is different: it is the closure of a finite time interval times a bounded convex developed spatial tube U, with $\overline{U} \cap \mathscr{W} = \varnothing$. All initial carriers, blanks, programme words, controller registers, and ancillary inputs are declared there. There is no undeclared incoming boundary data.

This is not an initial-boundary-value problem for a field. A meridian surrounding the defect needs a developing atlas; a single regular laboratory chart need not and does not contain the apex.

Complete channels and intervention-based influence

For finite input factors indexed by X, write

$$\mathscr{H}_X = \operatorname{bigotimes}_{x \in X} \mathscr{H}_x, \qquad \mathscr{A}_X(S) = \mathscr{B}(\mathscr{H}_S) \otimes I_{X \setminus S}.$$

Classical registers have a distinguished basis and diagonal observable algebra. Their canonical matrix extension first pinches that basis. Every controlled branch map is fixed independently of the unknown input density operator.

A channel is completely positive and trace preserving (CPTP). We use

$$d_{\mathrm{tr}}(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1, \qquad d_{\diamond}(\Phi, \Psi) = \frac{1}{2}\|\Phi - \Psi\|_{\diamond}.$$

Standard finite-channel conventions may be found in [7]. Instruments retain their complete records [8].

Definition (Reference-assisted signalling). For input region A, output region B, and $\Phi_B = \mathrm{Tr}_{B^c} \Phi$, define

$$\mathrm{Sig}_{\Phi}(A \to B) = \sup_{E, \rho, \mathscr{I}_A, \mathscr{J}_A} d_{\mathrm{tr}}\big((\Phi_B \otimes \mathrm{id}_E)(\rho_{\mathscr{I}}), (\Phi_B \otimes \mathrm{id}_E)(\rho_{\mathscr{J}})\big),$$

where E is any finite untouched reference, ρ is any input-reference state, and

$$\rho_{\mathcal{J}} = (\mathcal{I}_A \otimes \mathrm{id}_{A^cE})(\rho).$$

The interventions $\mathcal{I}_A, \mathcal{J}_A$ are deterministic CPTP maps on the same interface. They do not act on E.

Thus $0 \le \mathrm{Sig}_\Phi \le 1$. Postselected normalization is not an intervention in this definition.

Theorem (Exact no-signalling criterion). *For a finite channel Φ, the following are equivalent:*

1. $\mathrm{Sig}_\Phi(A \to B) = 0$*;*

2. *a channel* $\Xi : \mathcal{B}(\mathcal{H}_{A^c}) \to \mathcal{B}(\mathcal{H}_B)$ *satisfies* $\Phi_B = \Xi \mathrm{Tr}_A$*;*

3.

$$\Phi_B^*(\mathcal{B}(\mathcal{H}_B)) \subseteq I_A \otimes \mathcal{B}(\mathcal{H}_{A^c}).$$

The factorization holds with every untouched reference.

Proof. For (ii), local trace preservation leaves the A^cE reduction unchanged, proving (i). Conversely, compare the identity intervention with the local replacement

$$\mathcal{R}_A(Z) = \mathrm{Tr}(Z)\frac{I_A}{\dim \mathcal{H}_A}.$$

Condition (i), even without a reference, implies $\Phi_B = \Phi_B(\mathcal{R}_A \otimes \mathrm{id})$ on all states and therefore on all operators. Set

$$\Xi(Z) = \Phi_B\left(\frac{I_A}{\dim \mathcal{H}_A} \otimes Z\right).$$

It is CPTP and proves (ii). Taking adjoints gives (iii). Conversely, (iii) makes every output expectation depend only on the A^c reduction; the same replacement argument gives (ii). Tensor the factorization with id_E.

This is the precise finite criterion used in VII-J [1, thm:nosignal]. It does not identify correlation with signalling.

Example (A joint output can reveal a signal hidden in marginals). Encode a classical input bit b as $(R, R\mathrm{mathbin} \oplus b)$, with R uniform. Each individual share is independent of b, but their joint parity equals b. Separate marginal tests cannot replace a joint-output certificate.

Example (Postselection does not refute deterministic locality). For an entangled pair, selecting a local measurement outcome can change a conditional remote state. Summing the outcomes leaves the remote marginal unchanged. The latter is the deterministic comparison in Definition ; an outcome used remotely requires a declared classical wire.

The complete timed network and exact scheduled support

Definition (Complete finite network). A complete network is a finite directed acyclic circuit of CPTP maps, including:

• quantum and classical carrier ports and identity wires;

• fixed independent ancillary preparations;

• gates, storage, allocations, resets, and discards;

• destructive measurements and all retained outcomes;

• every controller variable read to choose a gate, guard, route, reset, or availability time;

• the encoder and decoder, with their actual support. A finite adaptive instruction tree is padded to a finite common envelope containing every branch and its selecting controls. There is no hidden state-functional scheduler.

For a conservative dependency graph, connect every input of a gate to every output. A reduced dependency graph is admissible only if it has a proved support statement for *every joint subset* of that gate's outputs, including references. Single-output no-signalling alone is insufficient.

For joint terminal output B, let $\mathrm{Sch}^{-}(B)$ be the model input factors with a directed dependency path to B.

Theorem (Complete scheduled-support bound). *For the channel Φ of a complete network,*

$$\Phi^{*}(\mathscr{A}_{Y}(B)) \subseteq \mathscr{A}_{X}(\mathrm{Sch}^{-}(B)).$$

Inputs outside $\mathrm{Sch}^{-}(B)$ *cannot signal to B, including when the observer also retains an arbitrary untouched*

reference. Replacing a gate outside the backward region by any CPTP map on the same ports does not change that output reduction.

Proof. Pull an output observable backward in reverse topological order. A gate whose outputs all carry identity is invisible, because its adjoint is unital. Otherwise the conservative graph adds only that gate's input ports. Certified reduced graphs use their joint-output support hypothesis instead. A fixed preparation creates no unknown model-input predecessor. Measurements, classical feed-forward, and resets are ordinary complete channel nodes in this argument. Induction gives . Apply Theorem . The same reasoning with reference observables proves complete equality. On an excluded gate the pulled-back observable is identity, also preserved by any replacement channel.

This reproduces the support mechanism of VII-J and the partitioned-channel comparisons of En03-A and En05-A [1].

Corollary (Partitioned metric envelope). *For layers* $\Phi_j = \mathrm{bigotimes}_{B \in \mathscr{B}_j} \Phi_{j,B}$ *on a finite metric site set* (X, d_{comp}), *assume* $\operatorname{diam} B \le \Delta$. *Define backward sets by*

$$N_{t,t}(Y) = Y, \qquad N_{s,t}(Y) = \bigcup_{\text{substack } B \in \mathscr{B}_{s+1}}$$

Then

$$\operatorname{supp}(\Phi^*_{s+1} \cdots \Phi^*_t(O_Y)) \subseteq N_{s,t}(Y), \qquad N_{s,t}(Y) \subseteq \{x : d_{\mathrm{comp}}(x, Y) \le (t-s)\Delta\}.$$

Proof. The support assertion is Theorem . Each backward layer adds at most one block diameter. Apply the triangle inequality inductively.

The radius is $(t-s)\Delta$, not $t-s$ without the normalization $\Delta = 1$. The computational metric is an input; it is not inferred from the conical metric.

Example (Strictness of the first distinction). Two successive SWAPs have a diagram with cross-register dependencies but total channel id. The first input has no influence on the second output. Thus scheduled ancestry can strictly overestimate actual influence.

Proposition (What retiming preserves). *Swapping adjacent independent channel nodes acting on disjoint factors preserves the complete channel, including records and references. It need not preserve an assigned physical timing certificate.*

Proof. The two tensor-factor maps commute, also after amplification. A finite sequence of such swaps preserves their composition. Their new assigned port times need not satisfy the original causal inequalities, which are separate hypotheses.

Logical depth, hardware execution time, slicing parameter s, geometric time T, and clock-code labels are not identified.

Causal embedding, realized routes, and robustness

Give each port v a prescribed event $p_v \in M$. Every dependency edge $u \to v$ must have a certified future-causal route in the operational tube. For uncertain regions L_u, L_v, a conservative sufficient condition is

$$p \operatorname{preceq}_g q \quad \text{for all} p \in L_u,\ q \in L_v.$$

A weaker allowance must specify the correlated endpoint choices and their actual routes.

Theorem (Encoded physical containment). *Suppose the full encoded–computed–decoded network is complete, and every dependency edge has the stated causal certificate. For a joint output B, let* $\mathrm{Phys}^{-}(B)$ *contain input factors whose assigned events can reach at least one output event of B. Then*

$$\mathrm{Inf}^{-}_{\Phi}(B) \subseteq \mathrm{Sch}^{-}_{\mathrm{full}}(B) \subseteq \mathrm{Phys}^{-}(B).$$

Every intervention supported outside $\mathrm{Phys}^{-}(B)$ *has zero signalling to B, with arbitrary untouched references.*

Proof. Theorem gives the first inclusion. Concatenate the supplied causal routes along a scheduled path. Transitivity gives the second inclusion. Apply Theorem to the complement of the physical past.

The distinction in is the central VII-J certificate [1, thm:physical]. A physically possible path with no installed wire shows that the second inclusion need not be equality.

Admitted implementation primitives

Assumption (Finite routing architecture). The model permits exact carriage of a finite register along a specified future-causal curve; positive-duration local gates and complete instruments on co-located registers; local independent preparation; local storage, resets, and discards; and classical records carried on explicitly scheduled causal wires.

These are ideal finite-model permissions, not a field-theoretic or laboratory construction of the communication mechanism.

Proposition (Gather–operate–distribute). *Suppose all gate inputs have supplied causal routes to one station event r_-, all outputs have supplied routes from a later station event r_+, and a finite local implementation fits between r_- and r_+. Then Assumption realizes that gate, with its records, ancillary factors, and references, as Target B.*

Proof. Carry the input factors to the station, execute the stipulated local channel, and distribute its outputs. Identity carriage and the local channel give the intended complete composition on all operators. Every elementary dependency has an explicit causal route.

Pairwise causal input–output possibilities alone do not establish this common-station hypothesis.

Proposition (Composition through a reindexing or event). *If $\Phi = \Phi_+ \mathscr{R} \Phi_-$ and the three channels have predecessor maps $P_+, P_{\mathscr{R}}, P_-$, then*

$$\Phi^*(\mathscr{A}(B)) \subseteq \mathscr{A}(P_-(P_{\mathscr{R}}(P_+(B)))).$$

Causal certificates compose in the same order. Independent allocations have no model-input predecessors; discards and all remapping controls must be included.

Proof. Compose the adjoint inclusions and then the corresponding causal paths.

The conservative slicing-parameter speed

All spatial coordinates in the next statements are dimensionless. Let a regular spatial route in the supported presentation have length D.

Theorem (Realized-route cone certificate). *A spatial route parametrized over $s \in [a, b]$, with metric speed at most*

$$v_* = \frac{1}{1+\lambda} = \frac{1}{1+2\sqrt{2}/3},$$

lifts under $s \mapsto (F_s(x(s)), x(s))$ to a future-causal curve. In particular, if an actual regular route of length D joins x to y, then

$$b - a \geq (1+\lambda)D$$

is sufficient for a causal route between its corresponding slice ports. The statement holds across $s = 0$.

Proof. Lemma gives, for an increment $\delta s > 0$,

$$F_{s+\delta s}(x(s+\delta s)) - F_s(x(s)) \geq \delta s - \lambda\, d(x(s+\delta s), x(s)).$$

The lift is Lipschitz. At almost every parameter its time derivative is at least $1 - \lambda|\dot{x}|$. When $|\dot{x}| \leq v_*$, this is at least $|\dot{x}|$, so the lifted tangent is future causal. For the final assertion, traverse the supplied spatial route at constant arc-length speed $D/(b-a)$, allowing waits.

The source's conservative speed is retained [1, cor:conespeed]. It is an inner sufficient bound in the slicing parameter, not a derived invariant speed or the full physical velocity set.

Remark (Distance equality on the punctured cone). A spatial distance defined as an infimum need not be realized by a path in the regular part: the limiting minimizer could meet the removed apex. Consequently a statement at exact distance equality needs a realized regular path. Our benchmark supplies such paths in a convex regular chart.

Proposition (Robust port margin). *In a convex regular developed chart, let a nominal slice-time gap be h, nominal spatial endpoint distance at most D, spatial localization radii* r_-, r_+*, slicing-time errors at most* ε_s *at each endpoint, and additional dimensionless geometric-time errors at most* ε_τ *at each endpoint. All endpoint choices remain causally admissible if*

$$h - (1 + \lambda)D \geq (1 + \lambda)(r_- + r_+) + 2\varepsilon_s + 2\varepsilon_\tau.$$

Strict inequality gives positive slack.

Proof. The actual spatial distance is at most $D + r_- + r_+$, and the actual slicing gap is at least $h - 2\varepsilon_s$. The two-family estimate gives actual geometric-time difference at least

$$h - 2\varepsilon_s - \lambda(D + r_- + r_+) - 2\varepsilon_\tau.$$

Under it exceeds the actual spatial distance. The straight developed segment is available within the chart.

This is an availability certificate, not permission to replace an exact event guard by a rounded geometric report.

Directional reachability and finite-time certificates

These are kinematic graph statements. An arbitrarily long periodic lift is not asserted to be embedded in the one bounded conical laboratory. Only finite prefixes supplied with actual routes enter Theorem .

Cycle velocities with a transient remainder

Let Q be a finite strongly connected directed multigraph. Its vertices include schedule phase and register type. Edge e has displacement $\Delta_e \in \mathbb{R}^d$ and duration $t_e \geq t_{\min} > 0$. The translation lift permits the declared copies of these transitions. Waiting is permitted in a manner compatible with the phase model.

For each simple directed cycle C, define

$$v(C) = \frac{\sum_{e \in C} \Delta_e}{\sum_{e \in C} t_e}, \qquad K_Q = \operatorname{conv}(\{0\} \cup \{v(C) : C \text{simple}\}).$$

This is the timed version of the periodic-graph velocity construction [9, 1].

Theorem (Finite-time offset and limiting velocities). *Let* $q = |V(Q)|$, $D_{\max} = \max_e |\Delta_e|$, *and* $R_Q = (q-1)D_{\max}$. *Every path completed within elapsed time* t *has displacement*

$$\Delta_{\text{path}} \in tK_Q + \overline{B}(0, R_Q).$$

From any specified initial quotient vertex, the set of subsequential limiting displacement/time ratios, for unbounded elapsed times, is exactly K_Q.

Proof. Remove closed subwalks until the remaining walk has no repeated vertex. Decompose removed closed walks into simple cycles. Their total displacement is a time-weighted sum of cycle velocities, with total cycle time at most t. The zero vector accounts for unused time. The remaining simple walk has at most $q-1$ edges and displacement norm at most R_Q, proving .

Divide by t to obtain one inclusion for limiting ratios. Conversely, take any convex combination of finitely many cycle velocities and zero. Repeat the cycles in integer counts whose durations approximate the chosen proportions on a scale tending to infinity. Strong connectivity supplies fixed connecting walks from the chosen start and between cycles. Their total cost is bounded for this fixed combination. Waiting realizes its zero component. The normalized displacement tends to the desired combination.

For a graph with several strongly connected components, only cycles that can be visited in the allowed component order may be combined. The convex hull of every cycle can overestimate attainability from a specified start.

A finite potential certificate

Proposition (Directional transient potentials). *Fix* $n \in \mathbb{R}^d$ *and* $\beta \geq 0$. *The following are equivalent:*

1. $n \cdot v(C) \leq \beta$ *for every simple cycle;*

2. *finite potentials* π_v *exist such that*

$$n \cdot \Delta_e - \beta t_e \leq \pi_{\text{head}(e)} - \pi_{\text{tail}(e)} \quad (e \in E(Q)).$$

Every path from u *to* v, *of duration at most* t, *then satisfies*

$$n \cdot \Delta_{\text{path}} \le \beta t + \pi_v - \pi_u.$$

Proof. Summing around a cycle proves (i). Conversely, set $w_e = n \cdot \Delta_e - \beta t_e$. Every closed walk has nonpositive weight by cycle decomposition. Add a root with zero-weight edges to all vertices. Let π_v be the maximum weight of a root-to-v walk. Removing nonpositive closed subwalks does not decrease its weight, so a maximum exists among finitely many simple walks. Appending an edge gives $\pi_{\text{head}} \ge \pi_{\text{tail}} + w_e$, which proves (ii). Summation gives ; use $\beta \ge 0$ for a path completed before t.

This is a finite difference-inequality witness, not a new general graph-theoretic principle. It supplements the cycle polytope by explicit endpoint-dependent transient costs. For a physical directional bound $b(n)t$, the sufficient endpoint check is

$$(b(n) - \beta)t \ge \pi_v - \pi_u.$$

A constant potential would instead certify every edge in that direction directly. Neither potential feasibility nor a cycle average supplies the missing physical routes.

Example (Zero asymptotic velocity does not prevent a fast first hop). Two quotient states have transitions $+L$ and $-L$, each of duration one, and waits. Every cycle velocity is zero. The first hop still travels L in one unit. For a prescribed speed $c_0 < L$, it is unsafe. The finite offset or potential difference detects the transient.

Safety, coverage, and support functions

For compact convex $K \subset \mathbb{R}^d$, define $h_K(n) = \sup_{v \in K} n \cdot v$.

Proposition (Directional inclusion). *For nonempty compact convex sets K, L,*

$$K \subseteq L \quad \Leftrightarrow \quad h_K(n) \le h_L(n) \text{ for all} n.$$

If L is a polytope, its defining facet normals suffice. If true support functions have certified errors $\delta_K(n), \delta_L(n)$, the nominal gap $h_L(n) - h_K(n) \ge \delta_K(n) + \delta_L(n)$ certifies true inclusion.

Proof. Inclusion implies the inequalities. If $v \in K \setminus L$, choose its nearest point $z \in L$. Then $(v - z) \cdot (w - z) \le 0$ for

all $w \in L$, which strictly separates v from L. The facet and error statements follow by substitution.

For cardinal hops of length a, one per interval δ, or a wait, in two dimensions,

$$K_{\mathrm{Sch}} = \{v : |v_1| + |v_2| \le a/\delta\} \subseteq K_{\mathrm{phys}} = \{v : |v|_2 \le a/\delta\}.$$

Their support functions are $(a/\delta)\|n\|_\infty$ and $(a/\delta)\|n\|_2$. The disk is not contained in the diamond: $(a/(\sqrt{2}\delta), a/(\sqrt{2}\delta))$ is a counterexample. Thus the diamond is safe but does not cover all physical directions.

If a constant physical velocity set K_{phys} contains $K_Q + \overline{B}(0, R_Q/t)$, then certifies endpoint reachability within time t. It does not certify the intermediate implementation paths. For Target B, check the actual route velocities locally.

Earliest arrivals and coordinate-varying examples

On a finite time-expanded DAG, earliest arrivals are obtained by the recurrence

$$A(v) = \min_{u \to v} \{\text{allowed arrival at} v \text{after arrival} A(u)\},$$

including all waits and availability restrictions. Topological induction proves the recurrence exactly. The value is $+\infty$ when no path exists.

Two closed-form comparisons can be made without changing the prescribed local Minkowski geometry. In a one-dimensional regular rod coordinate x, the time reparametrization $\tau = a\log(1 + u)$, $a > 0$, gives

$$\left|\frac{\mathrm{d}x}{\mathrm{d}u}\right| \le \frac{a}{1+u}.$$

Therefore

$$|y - x| \le a\log\frac{1+u_1}{1+u_0}, \qquad u_{\mathrm{arr}} = (1+u_0)e^{|y-x|/a} - 1.$$

This is time-varying, not spatially varying.

For a genuinely spatially varying coordinate speed, set $x = a\log(1 + q)$, $q > -1$. Then

$$|\dot{q}| \le \frac{1+q}{a}, \qquad \tau_{\mathrm{arr}} = \tau_0 + a\left|\log\frac{1+q_1}{1+q_0}\right|.$$

Both formulas follow by integrating the rod-distance bound; monotone null motion attains equality. They are coordinate descriptions of the installed regular geometry, not derived gravitational dynamics.

Approximate simulation and the Hamiltonian comparison

Theorem (Signalling stability with consistent constants). *For channels on the same interfaces,*

$$\left|\mathrm{Sig}_{\Phi}(A \to B) - \mathrm{Sig}_{\Psi}(A \to B)\right| \le \left\|\Phi_B - \Psi_B\right\|_{\diamond} = 2d_{\diamond}(\Phi_B, \Psi_B).$$

In particular, if Ψ is nonsignalling and $d_{\diamond}(\Phi_B, \Psi_B) \le \varepsilon$, then $\mathrm{Sig}_{\Phi} \le \min\{1, 2\varepsilon\}$.

Proof. For fixed reference, state, and interventions, the two comparison errors each have trace norm at most $\left\|\Phi_B - \Psi_B\right\|_{\diamond}$. The triangle inequality and the 1/2 trace-distance factor give one such full-diamond norm for the difference of the two distinguishabilities. Take suprema and exchange the channels.

This retains VII-J's norm convention [1, thm:signal-stability]. An expectation bound for one state or one observable does not supply its diamond-norm hypothesis.

Proposition (Complete-channel error ledger). *For compatible sequences of channels,*

$$d_{\diamond}(\widetilde{\Phi}_N \cdots \widetilde{\Phi}_1, \Phi_N \cdots \Phi_1) \le \min\left\{1, \sum_{j=1}^{N} d_{\diamond}(\widetilde{\Phi}_j, \Phi_j)\right\}.$$

Every encoding, clock, controller, transition, communication, decoding, and failure output must be compared on matching interfaces.

Proof. Replace one factor at a time. The resulting telescoping sum has one channel difference in each term. Composition with channels contracts the diamond norm. Use the triangle inequality and divide by two.

Corollary (Accuracy and causal safety are independent obligations). *A causally embedded implementation has exact zero signalling outside its certified physical past, irrespective of its accuracy. If its target has signalling s_0 on a forbidden pair, any half-diamond approximation to that target has error at least $s_0/2$. Conversely, an arbitrarily accurate approximation to a nonsignalling target need not be exactly nonsignalling.*

Proof. Use Theorems and . For the last assertion,

$$\Phi_{\varepsilon} = (1 - \varepsilon)\mathscr{R}_{0} + \varepsilon \mathrm{id}$$

on a qubit differs from the constant channel $\mathscr{R}_{0}$ by half-diamond distance at most ε, but has signalling ε between its input and output.

A finite-volume influence tail

Let $G = (X, E_{G})$ be finite and

$$H(t) = \sum_{x} h_{x}(t) + \sum_{e \in E_{G}} h_{e}(t), \qquad \Lambda(s,t) = \frac{1}{\hbar}\int_{s}^{t} \sum_{e} \| h_{e}(r) \| \, \mathrm{d}r,$$

with bounded piecewise-continuous self-adjoint onsite and two-site edge terms. Let $\Psi_{t,s}$ be its unitary channel.

Theorem (Reference-stable finite-volume tail). *For disjoint nonempty A, B, let $d = d_{G}(A, B) \geq 1$. Then*

$$\mathrm{Sig}_{\Psi_{t,s}}(A \to B) \leq \min\left\{1, \sum_{m=d}^{\infty} \frac{(2\Lambda(s,t))^{m}}{m!}\right\}.$$

For every $\gamma > 0$, the series is at most

$$\exp\left(-\gamma d + 2e^{\gamma}\Lambda(s,t)\right).$$

The estimate is independent of the untouched reference dimension, but is not uniform in the number of edges.

Proof. Pass to the interaction picture for the onsite propagator, a product of onsite unitaries. It preserves support and all edge-term norms.

For a Hermitian output observable O_{BE} of norm at most one, expand the Heisenberg evolution in its norm-convergent ordered nested-commutator series. At order m, summing edge choices and ordered integrals bounds the norm by $(2\Lambda)^{m}/m!$. Each nonzero commutator must meet the current site support, and can extend its graph distance from B by at most one. Every term of order below d is therefore identity on A, also when its coefficients are arbitrary operators on E.

The two intervention adjoints agree on that partial sum. They are operator-norm contractions, so their difference on the remainder is bounded by twice the series in . Trace-norm duality and the 1/2 normalization in Definition

give . Insert $1 \le e^{\gamma(m-d)}$ for $m \ge d$ and sum the exponential series for the final bound.

This is the conservative finite-volume route of VII-J [1, thm:tail]. Sharper quasi-local estimates require additional interaction hypotheses [10, 11]. We do not claim a new Lieb–Robinson theorem.

Example (A Hamiltonian tail is not an exact light cone). For two assigned separated sites, take $H = JX_A X_B$, $J > 0$. Prepare B in $|0\rangle$, and replace A by $|+\rangle$ or $|-\rangle$. Their output states on B are $e^{\mp iJtX/\hbar}|0\rangle$, with trace distance

$$|\sin(2Jt/\hbar)|.$$

It is nonzero for arbitrarily small positive t. Such a Hamiltonian cannot be identified with an exact geometrically local communication primitive at positive separation.

Constructive finite simulation estimates

Proposition (Duhamel and product-formula bounds). *For bounded self-adjoint generators on a fixed finite space,*

$$d_\diamond(\mathrm{Ad}_U, \mathrm{Ad}_{\widetilde{U}}) \le \|U - \widetilde{U}\| \le \frac{1}{\hbar}\int_s^t \|H(r) - \widetilde{H}(r)\|\,\mathrm{d}r.$$

For time-independent $H = \sum_{j=1}^m H_j$, put

$$V_n(t) = \left(e^{-itH_m/(n\hbar)}\cdots e^{-itH_1/(n\hbar)}\right)^n, \qquad C_H = \sum_{j<k}\|[H_j, H_k]\|.$$

Then

$$\left\|e^{-itH/\hbar} - V_n(t)\right\| \le \frac{t^2 C_H}{2n\hbar^2}, \qquad d_\diamond(\mathrm{Ad}_{e^{-itH/\hbar}}, \mathrm{Ad}_{V_n(t)}) \le \min\left\{1, \frac{t^2 C_H}{2n\hbar^2}\right\}.$$

Proof. Differentiate the mixed propagator $U(t,r)\widetilde{U}(r,s)$, integrate, and use unitarity. The channel comparison follows by expanding $UXU^\dagger - VXV^\dagger$ into two terms, also after amplification.

For two summands, the double integral comparing the joint exponential and ordered product has norm at most $\delta^2\|[H_1, H_2]\|/(2\hbar^2)$. Iterating over the finite sum gives one-step error $\delta^2 C_H/(2\hbar^2)$. Telescoping n unitary steps with

$\delta = t/n$ proves .

The product-formula channel is a logical approximation. Its elementary implementation has its own positive travel and gate times. Approximating a Hamiltonian to accuracy does not embed its numerical clock into the physical clock.

Destructive clocks, programme records, and calibration

Let $C \cong \mathbb{C}^{2^m}$ have an installed orthonormal basis $\{|\tau_a\rangle\}$. The surviving space S includes matter and every retained control or location/history register. Set

$$P_a = |\tau_a\rangle\langle\tau_a|, \qquad D_a = \langle\tau_a|_C \otimes I_S.$$

Theorem (Complete destructive-clock instrument). *The basis change $|\tau_a\rangle \mapsto |a\rangle$, followed by destructive Enter on all clock names, has channel*

$$\mathscr{M}_C(X) = \sum_a \operatorname{proj} a_R \otimes D_a X D_a^\dagger = \sum_a \operatorname{proj} a_R \otimes \operatorname{Tr}_C[(P_a \otimes I)X(P_a \otimes I)].$$

It is CPTP on all operators, including every reference extension. Its output has no quantum C factor, and

$$\operatorname{Tr}_R \mathscr{M}_C(X) = \operatorname{Tr}_C X.$$

For branch probability $p_a > 0$, the conditional surviving state is $D_a \rho D_a^\dagger / p_a$. Zero-probability branches are not normalized.

Proof. Kraus operators are $|a\rangle_R \otimes D_a$, with squared moduli summing to $\sum_a P_a \otimes I = I$. Expansion in the clock basis shows that both expressions in return its aa block. Summation gives the partial trace. The calculation is unchanged after adjoining an arbitrary untouched factor. The destructive output type removes the measured quantum names.

This is the I-B/En09-A finite interface [2, 21, 22]; its conditional block is not a metric, stress tensor, or singular detector.

For fixed continuation channels $\mathscr{F}_a$, the full programme channel is

$$\mathcal{J}(X) = \sum_a \mathrm{proj}\, a_R \otimes \mathcal{F}_a(D_a X D_a^\dagger).$$

It is CPTP because each continuation preserves the trace of its unnormalized block. Routing choices and record delivery must be implemented before the receiver uses a. An arbitrary state-functional selection of $\mathcal{F}_a$ is not this channel.

Proposition (Calibration boundaries). *Let distinct numbers θ_a label clock codes. At one actual reading event e, exact event-time calibration*

$$p_a > 0 \Rightarrow \theta_a = T(e)$$

forces a single deterministic outcome. If no label equals $T(e)$, it is impossible for a normalized input.

A history or programme dictionary may instead associate a with a different labeled event, arrival, route, or stored incidence parameter. It need not equal the actual reading time.

Proof. Distinctness allows at most one matching index and the probabilities sum to one. The dictionary interpretation makes no claim that several different labels are coordinates of the same reading event.

A programme label for a future arrival is a planned target, not evidence that the arrival already occurred. A history claim requires its independently specified preparation and provenance.

Proposition (No universal quantum return or finite CCR). *The channel $\mathcal{M}_C$ is not invertible on all clock inputs. Finite matrices cannot satisfy $[T_C, H_C] = i\hbar I$ when $\hbar \neq 0$. Consuming C does not remove T, M, or g.*

Proof. The clock states $(|\tau_0\rangle \pm |\tau_1\rangle)/\sqrt{2}$, with the same payload, have identical outputs. A left inverse is impossible. Taking the trace proves the commutator assertion. The final statement follows because these geometric objects are prescribed parameters, not output factors of $\mathcal{M}_C$.

Optional relational interpretation

For completeness, the additional hypothesis needed to turn conditioning into relational evolution can be stated precisely [12, 21, 22].

Proposition (Sampled covariance and stationarity). *Suppose finite Hermitian H_C, H_S and ρ_{CS} satisfy*

$$[H_C \otimes I + I \otimes H_S, \rho] = 0, \qquad P_a = U_C(t_a) P_0 U_C(t_a)^\dagger.$$

Then

$$D_a \rho D_a^\dagger = U_S(t_a)(D_0 \rho D_0^\dagger) U_S(t_a)^\dagger.$$

For a complete orthonormal reading basis all probabilities equal 2^{-m}. For an extended SE assertion, stationarity of the full extension is required.

Proof. Stationarity gives

$$(U_C^\dagger \otimes I)\rho(U_C \otimes I) = (I \otimes U_S)\rho(I \otimes U_S^\dagger).$$

Insert the covariant projector into the clock partial trace and use unitary invariance of that trace. The resulting blocks are unitarily related, so their traces are equal and sum to one. The same calculation applies to an extension exactly under the extended stationarity hypothesis.

No such dynamical covariance is assumed for the programme table below. The external schedule remains an input.

Records and residuals remain different certificates

Two record-retaining instruments may have the same outcome law and different residuals. For example, with one coarse outcome, the identity qubit channel and computational dephasing have identical records but, on proj +, output proj + and $I/2$. Their trace distance is 1/2. Implementation errors $\varepsilon_1, \varepsilon_2$ leave a gap at least $1/2 - \varepsilon_1 - \varepsilon_2$, by the triangle inequality. This is the observation-boundary discipline compared with VII-E, VII-G, and VII-E-ST [14].

More generally, a complete recorded instrument that recovers every input in the full quantum algebra can have only input-independent record effects. Indeed, for Kraus operators A_i of the instrument and B_j of a proposed recovery, all $B_j A_i$ are scalar multiples of I, because the identity channel has rank-one Choi matrix. Completeness of the recovery then gives $A_i^\dagger A_k = \lambda_{ik} I$. Summing the diagonal terms for one record makes its effect scalar. This

full-input criterion is consistent with standard quantum error correction [13, 7]. It does not prohibit transport of the surviving payload.

Exact regular-part benchmark through the incidence parameter

We now give a nonempty Target B instance. It deliberately replaces VII-J's singular-axis rendezvous [1, thm:eventbenchmark].

Persistent spatial marks and exact times

In sector zero choose the dimensionless transverse mark

$$x_{\perp} = (1/64, 1/128).$$

At that mark,

$$h(x_{\perp}) = 1/64, \qquad k(x_{\perp}) = 3/640.$$

Choose two regular persistent ports

$$A = (x_{\perp}, \zeta = 0), \qquad B = (x_{\perp}, \zeta = L), \qquad L = \frac{1}{64},$$

and slice parameters

$$s_{-} = -\frac{1}{4}, \qquad s_{+} = \frac{1}{4}.$$

Both ports have identical time coordinates on each marking. Their input and output times are

$$\tau_{\mathrm{in}} = -\frac{15}{64}, \qquad \tau_{\mathrm{out}} = \frac{323}{640}.$$

Take a convex developed spatial tube containing

$$[3/256, 5/256] \times [1/256, 3/256] \times [-1/256, 5/256].$$

Its closure is entirely regular and lies inside the first sector. Choose a finite time interval containing $[\tau_{\mathrm{in}}, \tau_{\mathrm{out}}]$. All operational paths and gates remain in this tube.

The marks A, B are spatial locations, not evolving abstract cell labels. The local branch at $x_{\perp}$ changes at $s = h - k = 7/640$, but the mark and its quantum register do not cease to exist.

A fixed finite event-code restriction

The complete VII-F metadata circuit is not silently inferred from these few bits. We use its following explicit fixed-instance restriction.

At the controller A, the exact stored incidence parameter is $j = 0$; the fixed geometric packet specifies the prescribed cone and zero germ. Reserve classical mode, permission, and history bits

$$(M, N, H) = (0, 1, 0),$$

and one scratch bit $E = 0$. A two-qubit computational-basis clock returns $a = (a_1, a_0) \in \{00, 01, 10, 11\}$. Only $a = 01$ requests the event update. Set

$$e(a) = (1 - a_1)a_0.$$

On the promised initial bits the update is

$$(M, N, H) = (0, 1, 0)\text{longmapsto}\,(e(a), 1 - e(a), e(a)).$$

Thus the canonical programme 01 performs the exact incoming-to-outgoing zero-germ mode update, spends its one permission, and retains a history bit. Other programmes request no update. No length report is used as a clock guard.

The permutation is implemented by:

1. NOT on a_1, Toffoli $(a_1, a_0; E)$, NOT on a_1;

2. CNOT from E to each of M, N, H;

3. the inverse of step 1. There are nine elementary NOT/CNOT/Toffoli gates. The record is restored and E returns to zero. Off the promised domain this remains a reversible bit permutation, but is not assigned a geometric interpretation.

Remark (A full supplied event circuit). Any separately specified finite source circuit U_{code} with k local code/history/work carriers and G gates can replace this restriction. Co-locate its carriers at A and give each gate a positive duration $1/(16(G+4))$ in the buffer below. This localizes the supplied finite map; it does not prove an unsupplied decoder theorem or its forward promise. The explicitly proved numerical ledger below concerns .

Target-adapted chains and the buffer

Fix an integer $n \geq 1$. Install a chain of qubit registers $Q_0, \ldots, Q_n$ at

$$(x_{\perp}, \zeta_j), \qquad \zeta_j = jL/n.$$

Here Q_0 is a temporary register at A; Q_n is the input register at B. A separate qubit P is the input at A. Initialize $Q_0, \ldots, Q_{n-1}$ in $|0\rangle$.

Let

$$D = \frac{4L}{v_*} = \frac{1+\lambda}{16} < \frac{1}{8}.$$

During the inbound interval of length D, use n successive content exchanges

$$(Q_n, Q_{n-1}), \ldots, (Q_1, Q_0),$$

each of duration D/n, realized by opposite causal carriage of the two contents along the regular axial segment. Their physical dimensionless speed is $v_*/4 < 1$.

The unknown qubit reaches A by

$$\tau_{\mathrm{in}} + D < -\frac{7}{64} < 0.$$

Both matter qubits then remain stored at A. Destroy the two clock qubits in two local slots of duration $1/64$, from $\tau = -1/32$ to 0. The complete record is available at $\tau = 0$.

Wait until $\tau = 1/16$. Run the nine event-code gates, three local CNOTs implementing SWAP_{P,Q_0}, and the classical-record controlled gate $Z_P^{a_1}$. The thirteen local slots each have duration $1/208$, so they finish at $\tau = 1/8$.

Finally move Q_0's content outward by the n successive exchanges

$$(Q_0, Q_1), \ldots, (Q_{n-1}, Q_n).$$

Allow duration D/n per outbound stage. Its first half copies the five-bit classical word

$$r_a = (a_1, a_0, e(a), 1 - e(a), e(a))$$

into the next site's blank record buffer by five sequential classical CNOTs. Each copy slot has positive duration $D/(10n)$. The second half carries the qubit exchange and the copied record over the segment of length L/n. The communication speed is $v_*/2 < 1$.

All records and payloads arrive by

$$\frac{1}{8} + D < \frac{1}{4} < \tau_{\text{out}}.$$

Wait at the destination ports until τ_{out}. Records at earlier sites are retained, not reset.

Lemma (Every benchmark operation is regular and causal). *The specified execution is a finite Target B schedule. The matter is buffered across both the worldsheet incidence time* $\tau = 0$ *and the local slice parameter* $s = 0$. *No operation meets* $\mathscr{W}$.

The moving routes also satisfy the sufficient slicing speed .

Proof. The entire axial chain is in the displayed regular tube. Every local operation is on one station worldline with strictly positive duration. The inbound exchange routes have speed $v_*/4$, and the outbound communication portions have speed $v_*/2$; both are future timelike.

The incoming branch has $\partial_s F_s = 1$. On the outgoing communication interval, $\partial_s F_s = 2$. Thus the route speeds in s are respectively $v_*/4$ and v_*. Stationary waits and gates have zero spatial speed. Theorem also certifies them.

The qubits are at A before $\tau = 0$, and remain there until $\tau = 1/16$. At A, the $s = 0$ slice has time $1/64$, which lies inside that storage interval. All control copies occur before their communication departures, and every remote use waits for the record to arrive.

The classical metadata describes the zero germ. Its positive-duration computation finishes later. It is neither an instantaneous global update of geometry nor acquisition of data from the singularity.

Complete channel and signalling witness

Write the input matter order as P, Q_n, and define

$$V_a = (Z^{a_1} \otimes I)\mathrm{SWAP}.$$

The programme $a = 01$ has $V_{01} = \mathrm{SWAP}$. The other programmes provide declared phase-controlled variants, not alternative spacetime metrics.

Theorem (Exact complete regular benchmark). *For every input operator X on $C \otimes P \otimes Q_n$, the complete output of the benchmark is*

$$\mathscr{J}_n(X) = \sum_{a \in \{0,1\}^2} (|r_a\rangle\langle r_a|)^{\otimes(n+1)} \otimes V_a D_a X D_a^\dagger V_a^\dagger \otimes \omega_{\mathrm{blank}},$$

where $D_a = \langle a|_C \otimes I_{PQ_n}$, and ω_{blank} contains the final blank $Q_0, \ldots, Q_{n-1}$ and scratch $E = 0$. The two matter output factors are at A, B, respectively. Equation holds with every untouched reference.

On the canonical clock preparation $|01\rangle\langle 01|$, the matter channel is exactly $\mathrm{Ad}_{\mathrm{SWAP}}$. It has

$$\mathrm{Sig}\,(A_{\mathrm{in}} \to B_{\mathrm{out}}) = 1.$$

No input outside the physically certified backward region signals to the declared joint output.

Proof. Successive exchanges with blank chain registers move the entire Q_n factor, with all its correlations, into Q_0 . The other chain registers become blank. The clock identity gives $D_a X D_a^\dagger$ on branch a. The nine-gate calculation gives . The three CNOTs exchange the local matter factors and the record-controlled phase gives V_a. Outbound exchanges move the second factor into Q_n, again returning all intermediate qubits to their initial blanks. The record is already classical; copying its specified basis word creates precisely the repeated projectors in , including their correlations with matter. No unknown quantum state is copied. This proves the formula on all operators and references.

For the witness, fix the clock to 01, hold the initial *B* qubit fixed, and replace the *A* qubit by |0⟩ or |1⟩. The *B* -outputs are orthogonal, hence their trace distance is one. Theorem gives the final no-signalling assertion.

The pre-existing reference can be entangled with the clock and both inputs. A conditional reference state may change at clock readout, but the channel acts as identity on the reference. No gravity-generated entanglement is claimed.

Proposition (No instantaneous exchange on one slice). *A zero-duration remote SWAP between distinct ports of a single regular marking cannot satisfy this causal architecture.*

Proof. Lemma makes the marking strictly spacelike: its time difference is bounded by λ times spatial path length. The ports are spacelike. A SWAP would have the unit intervention witness just used, contradicting physical containment.

An independent exact null-transport contract

The preceding buffer is deliberately timelike. Exact local null transport is also available under the I-B routing permission, but only for a supplied target-adapted chain.

Proposition (Target-adapted reference-preserving transport). *Let a developed segment of length* $L_0 > 0$ *remain in a convex regular chart. Install* $m + 1$ *equal finite payload registers along it and initialize all but the source in a fixed blank. Successive content exchanges over intervals* $L_0/(mc)$ *implement the logical identity at the null-related target, including every reference, using m exchanges.*

An already available classical record and finite location/history payload may be included in the transported packet. Nontrivial preparation, copying, or decoding time is additional.

Proof. At each hop the complete source factor replaces the next blank, and the previous site becomes blank. Induction proves the complete identity. Each communication path is the regular straight segment traversed at speed *c*. The same argument applies to a joint quantum–classical packet.

This is not an exact-attainment theorem for a previously fixed lattice or for records produced too late to enter the

packet.

Finite tables, resources, and checkable obligations

Finite evaluation table

For the benchmark's two-qubit clock, the exact table is

$c\|c\|c\|ca$	$e(a)$	$(M,N,H)_{\text{out}}$	V_a	$\mathrm{Sig}\,(A_{\text{in}} \to B_{\text{out}})$
hline 00	0	$(0,1,0)$	SWAP	1
01	1	$(1,0,1)$	SWAP	1
10	0	$(0,1,0)$	$(Z \otimes I)$SWAP	1
11	0	$(0,1,0)$	$(Z \otimes I)$SWAP	1

Each signalling entry is evaluated for that fixed programme. The full programme channel retains the record and is

.

The event indicator is the finite matrix

$$A_{\text{event}} = \mathrm{diag}\,(0,1,0,0).$$

A nominal label, route length, duration, cycle reward, or potential value can similarly be stored as a diagonal matrix over a finite programme set. Such matrices encode declared evaluations; they are not measurements of an unknown field.

A finite time-expanded dependency graph with N vertices can be represented by its Boolean adjacency matrix A. Its reachability table is the Boolean matrix

$$I \vee A \vee \cdots \vee A^{N-1}.$$

Every reachable pair has a path of at most $N-1$ edges in the DAG, which proves this finite test without numerical execution.

Quantum and classical resource ledger

For the explicit n-segment benchmark:

$l\|l$resource	sufficient reserve or count
hline matter and routing qubits	$n + 2$
clock qubits before destruction	2
quantum peak	$n + 4$
initial blank routing qubits	n
classical record/code buffers	$5(n + 1)$bits
event scratch	1bit, returned to zero
event-code elementary gates	9
local quantum CNOT gates	3
local record-controlled phase	1
outbound classical copying	$5n$CNOTs
destructive clock operations	2
qubit content exchanges	$2n$
classical record transmissions	n
resets during the run	0
retained record words	$n + 1$

The bits M, N, H are part of the five-bit sender word, not extra uncharged buffers. Their initial permission bit is one. History and record copies are retained outputs, not clean work.

For an explicit conservative finite controller reserve, put

$$b_n = \lceil \log_2 (8n + 32) \rceil.$$

Allow one b_n-bit local instruction counter per station and at most $8n + 32$ programme rows per station. An instruction row can be encoded with a four-bit opcode and three b_n-bit indices, so the programme-ROM reserve is

$$B_{\mathrm{ROM}} \le (n + 1)(8n + 32)(4 + 3b_n)$$

bits, in addition to $(n + 1)b_n$ counter bits. All station times are evaluated from the fixed algebraic schedule formulas, not acquired as unknown real parameters. A single remote global counter is not used as a hidden instantaneous control wire.

The ledger is sufficient rather than optimal. It assumes parallel carriage of the stated finite packet capacity on each admitted link. Different bandwidth, gate, or reset permissions require a new ledger. External reference storage is not charged as laboratory storage. No energy, thermal-machine, or Landauer-cost claim follows.

Exact geometric checks

The following inequalities are symbolic certificates, not reported numerical runs:

l|*l*obligation	witness
hline spacelike margin	$1 - \lambda^2 = 1/9$
conservative slice speed	$v_* = (1 + \lambda)^{-1}$
regular transverse port	$(1/64, 1/128) \neq o$
axial route length	$L = 1/64$
inbound duration	$D = (1 + \lambda)/16 < 1/8$
gather before clock readout	$-15/64 + D < -7/64 < -1/32$
local gate buffer	$13(1/208) = 1/16$
distribution before output	$1/8 + D < 1/4 < 323/640$
event scratch cleanup	$E_{\mathrm{out}} = 0$
event flags	$(M, N, H) = (e, 1 - e, e)$
canonical matter target	$V_{01} = \mathrm{SWAP}$

All nontrivial communication and local-operation intervals have positive causal slack. For any fixed finite n, sufficiently small port and timing errors preserve these checks by Proposition , or directly by the strict developed time-minus-distance margins. Shared input/output ports are the same physical event, not independently perturbed copies of an event.

Finite approximation of a fixed instrument

Suppose the benchmark's clock, local gates, record copies, and communication channels have half-diamond errors with sum $\varepsilon_{\mathrm{tot}}$ on their complete stated interfaces. Then Proposition gives

$$d_{\diamond}(\hat{\tilde{\mathscr{I}}}_n, \mathscr{I}_n) \leq \min\{1, \varepsilon_{\mathrm{tot}}\}.$$

The unit signalling witness on an allowed route remains at least $1 - 2\varepsilon_{\mathrm{tot}}$. Physical containment is exact only when the implemented dependencies retain their own causal certificates; channel accuracy alone supplies at most the leakage bound of Theorem .

No normalization of a rare selected branch is used in this error statement. The comparison is the complete recorded instrument.

Assembly, significance, and standing non-claims

Theorem (VII-J-ST local Grade S release). *For the declared conical geometry and finite permissions:*

1. *complete finite dependency graphs bound joint-output Heisenberg support and deterministic reference-assisted*

influence;

2. *certified causal routes yield* $\mathrm{Inf}^{-} \subseteq \mathrm{Sch}^{-} \subseteq \mathrm{Phys}^{-}$ *for the full encoded–computed–decoded channel;*

3. *directional cycle polytopes have the finite-time remainder , and the finite potential certificate records directional transients;*

4. *signalling stability has the constant* $2d_{\diamond}$*, and the finite Hamiltonian comparison has the reference-stable tail ;*

5. *destructive clocks realize the complete instrument , with event-time and programme/history calibrations distinguished;*

6. *the benchmark is an exact, nontrivial, reference-preserving Target B realization entirely in the regular part, buffered through the prescribed incidence parameter and supported by the displayed resource ledger. These statements constitute a nonempty local Grade S package.*

Proof. Theorems and prove (i). Theorem proves (ii). Theorem and Proposition prove (iii). Theorems and , with Proposition , prove (iv). Theorem and Proposition prove (v). Lemma , Theorem , and Section prove (vi). These provide all clauses of Definition .

What the adaptation contributes

The general support theorem, cycle-velocity mechanism, instrument calculus, and Hamiltonian norm methods have predecessors [1, 9, 8, 7, 10]. The adaptation does not claim priority for them.

Its contribution is their explicit supported assembly with:

• a regular-part replacement for the singular-axis benchmark;

• actual regular paths rather than an unrealized metric distance equality;

• complete records, controllers, and joint-output support;

• a finite transient-potential check in the directional ledger;

• an exact clock-controlled recorded channel and positive execution buffer;

• charged blank registers, history, record copies, local controllers, and programme storage. This combination is potentially useful as a test case for a later proposal to add field acquisition, continuum compatibility, or dynamical geometry. Those later proposals would need their own same-model interfaces and proofs.

Documentary source relationships

VII-E's block locality, legal retiming, record pullback, and residual obstruction are distinct source results; VII-G's finite hybrid-channel interfaces do not themselves supply a physical embedding [14]. En03-A and En05-A compare prescribed schedule support with separate metric envelopes [1]. En09-A distinguishes conditioning from its additional dynamical covariance hypotheses [21, 22].

VI-B and VII-A-ST supply supported-presentation comparisons, not imported Einstein–scalar equations or descent theorems. VIII-A/B/C distinguish preconstructed cone packets and source interfaces from gravitational dynamics. In particular,

$$\text{matter-controlled slicing} \neq \text{matter-driven gravitational dynamics.}$$

A location word is not the located quantum register. The fixed incidence changes a marking, not the background metric.

The OpenAI catalogue is a separate documentary collection attributed to OpenAI [15]. It supplies no geometry, channel, clock, formal premise, or verification of this manuscript. A documentary separation is not a proof transfer.

Version-specific analytical boundaries

The historical H2 ending in XIV-H is not discharged by this adaptation. It must be distinguished from XIV-I's constructed internal I2 amplitude, which is not the full characteristic map still absent at I3/J2. No internal amplitude or supported-sector reference is treated as the complete characteristic quantum composition.

The downstream quantum-admission exit called XV-B is not opened. This is not a claim that no technical manuscript with that identifier exists. XVI-N's universal obligations and Route B are not discharged. Within the

stated XVI-O prescription,

$$O.1 = \text{texttt detokenize}\, NOT_{C} LOSED.$$

Separately scoped enlarged-prescription subsector statements do not update that particular record. The quantum certificate remains RG-open; no quantum renormalization separator is proved.

par **Explicit non-claims** par No global conical Green/Cauchy theorem, intrinsic conical field theory, sourced Einstein–scalar solution, continuum-field acquisition, or quantum-field discretization is constructed.

No metric or Lorentz cone is derived from records or scheduler consistency. No Einstein dynamics, spacetime emergence, cosmological bounce, vacuum-energy stability, ultraviolet completion, Page curve, island, interior, or gravity-generated entanglement test is supplied.

No quantum Ward/BRST/BV identity, radiative coefficient, characteristic quantum response coefficient, or quantum evolution without external scheduling is admitted.

Exact finite channel equality is not a thermodynamic implementation. Clock consumption does not consume the geometric time axis. A buffered code update does not resolve the conical singularity. par

appendix

Suggested registry-facing record

The following is documentary text, not an assertion that an external registry or interface was changed.

identifier: VII-J-ST?grade: local Grade S?problemConnectionsFor: ["time"]?emergenceStatus: not constructed?quantumCertificate: RG-open??relationshipRole: separation?relationshipDistinction:? A documentary separation is not a proof transfer.??scope:? Complete finite dependency and causal-embedding certificates,? finite destructive clock control, and exact buffered regular? transport through one prescribed transverse incidence parameter.??corrections:? No apex quantum register is used.? The 3+1 defect is a worldsheet.? The 3-to-6 assertion is transverse.? All controllers and joint-output dependencies are included.? Half-diamond simulation

error contributes twice that amount? to the signalling bound.??standingBoundaries:? O.1 remains NOT_CLOSED in the stated XVI-O prescription.? The downstream XV-B quantum-admission exit is not opened.? XVI-N and Route B are not discharged.? Historical H2 and full I3/J2 records are not discharged.? No inherited quantum coefficient is admitted.

A suitable emergence-walk sentence is: VII-J-ST certifies finite scheduled influence and one buffered regular-part transport on a prescribed conical spacetime. It does not extract the causal cone, reconstruct the metric, or turn slicing incidence into gravitational dynamics.

Declarations

Anonymous review. The author field is intentionally empty. Authorized author, affiliation, and contribution metadata are required for a signed submission. Attribution of documentary records to N. Cheung or OpenAI does not assign authorship of this adaptation.

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Verification and implementation. The document is one conventional pdfLaTeX source with an internal bibliography. No successful pdfLaTeX execution, numerical run, symbolic-computation execution, hardware compilation, experiment, proof-assistant check, independent referee certification, or journal acceptance of this adaptation is asserted. The mathematical release consists of the stated definitions and proofs.

Book expansion notes (t433u)

Certified scheduled causality is not global hyperbolicity and not Einstein dynamics. Dedicated observation-boundary source record: [23].

Standing ledger. Not spacetime emergence; not Einstein dynamics; not a Page curve, island, bounce,

vacuum-energy selection, or de Sitter state as an Enter theorem; not a derived metric. O.1 stays NOT_CLOSED. XVI-N is not discharged. XV-B is not opened. No quantum coefficient is admitted. Host C is not created. Dom (T) remains undetermined; J2 remains unresolved. Problems 6, 7, and 9 are named, not solved. Gate 0 and ST4b stay NOT_SUPPLIED. Host-A stays NAMED_COMPARISON_ONLY $\{GI, CB\}$. The shredder is not Gibbons–Hawking. Cited sheets are source records, not premises. Conjectures stay conjectures. wired=false.

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